Summary of the recitation on 8. 1. 2008

We gave the proof of Van der Waerden's and Gallai-Witt's theorem. Then we worked on the following exercises:

- Let $K_{X,Y}$ be a complete bipartite subgraph with countably infinite parts X and Y. Is it true that for every two-coloring of the edges of $K_{X,Y}$ and for every n, the graph $K_{X,Y}$ has a monochromatic subgraph isomorphic to $K_{n,n}$? What about a monochromatic complete subgraph with one part of size n and the other part infinite? What about a monochromatic complete subgraph with both parts infinite? (Finished from last time)
- Show that a countable partially ordered set can be covered by k chains if and only if it has no antichain of size $k+1$. (Stated but not solved)
- Show that $HJ(k, 2) \leq k$.
- Consider the following "density version" of Hales-Jewett theorem:
  Let A be an alphabet of size ℓ. $\forall \varepsilon > 0\ \exists N \equiv N(\varepsilon, \ell)$ such that every set $X \subseteq A^N$ of size at least $\varepsilon \ell^N$ contains a combinatorial line.
  Show that this statement implies the ordinary Hales-Jewett theorem.