Summary of the recitation on 27. 11. 2007

We saw the following definitions:

- A graph G is a *k-tree* if and only if it can be obtained from a complete graph K_{k+1} by a sequence of steps, where in each step we add a new vertex and connect it to k vertices forming a clique of the original graph. A graph is a *partial k-tree* if it is a subgraph of a k-tree.
- A graph has *treewidth* equal to k if it is a partial k-tree but not a partial $(k-1)$-tree. The treewidth of G was denoted by $tw(G)$.
- An equivalent definition of treewidth may be obtained using the concept of *tree decomposition*. A tree decomposition of a graph $G = (V, E)$ is a tree $T = (B, F)$, where each vertex ("bag") $b \in B$ is a subset of V, and the following two properties are satisfied:
  1. For each vertex $v \in V$, the set $\{b \in B;\ v \in b\}$ induces a connected subtree in T.
  2. For every edge $\{u, v\} \in G$ there is a bag $b \in B$ containing both u and v.

  The *width* of a tree decomposition T is defined as $\max\{|b|; b \in B\} - 1$. The treewidth of a graph G may be defined as the smallest width of a tree decomposition of G. This definition is equivalent to the definition given above.

We then solved the following exercises:

- Show that the 1-trees are precisely the trees, and the partial 1-trees are precisely the forests.
- Show that each nonempty forest has a tree decomposition of width 1.
- Determine the treewidth of the complete graph K_n, the cycle C_n, and the complete bipartite graph $K_{m,n}$.
- Show that every partial k-tree has a vertex of degree at most k.

The following exercises were stated but not solved:

- Show that every k-tree has a tree decomposition of width k.
- Show that if G is a minor of H, then $tw(G) \leq tw(H)$.