## NMAG403 - Combinatorics

November 3, 2023 - Chromatic number and Chooseability

## Homework

Deadline: November 27, 2023
Send to: honza@kam.mff.cuni.cz (in PDF)

1. Prove that a tree has a perfect matching if and only if deleting any vertex creates exactly one component with an odd number of vertices.
2. Prove that the chooseability of any planar triangle-free graph is at most 4.
3. Let $G$ be a bipartite graph with $n$ vertices. Prove that $\operatorname{ch}(G) \leq\left\lceil\log _{2}(n)\right\rceil+1$.

## In class problems

27. Prove that every orientation of a bipartite graph has a kernel.
28. Prove that for every $k$, there is a triangle-free graph with $\chi(G)>k$.
29. Prove that for every $k$, there is a bipartite graph $G$ with $\operatorname{ch}(G)>k$.
30. Prove that the chooseability of any planar bipartite graph is at most 3 .
