NMAG403 - Combinatorics

October 27, 2023 – Matchings in graphs

In class problems

22. Prove the **Tutte theorem with defect**: For a positive integer d, a graph G contains a matching that misses at most d vertices if and only if for every set A of vertices of G, it holds true that

$$c_{odd}(G - A) \leq |A| + d.$$

(Hint: First prove the theorem for the case when d and $|V(G)|$ are of the same parity.)

23. Prove the **Edmonds Blossom Lemma**: Let M be a matching in a graph $G = (V, E)$ and let $C \subseteq G$ be a cycle of length $2k + 1$ in G which contains k edges of M and one free vertex (with respect to M). Let $\widetilde{G}$ be the graph obtained from G by contracting the cycle into one vertex, and let $\widetilde{M} = M \setminus E(C)$. Then M is a maximum matching in G if and only if $\widetilde{M}$ is a maximum matching in $\widetilde{G}$.

24. Design a polynomial time algorithm for constructing an Edmonds forest in an input graph.

25. The b-FACTOR problem is the problem to decide if an input graph G has a spanning subgraph H such that $\deg_H u = b(u)$ for every vertex $u \in V(G)$, where $b : V(G) \to \{0, 1, 2, \ldots\}$ is a function also given as part of the input. Show that b-FACTOR is polynomial time solvable.

26. Let BOUNDED-DEGREE-SUBGRAPH denote the problem which asks whether an input graph G has a spanning subgraph H such that $a(u) \leq \deg_H u \leq b(u)$ for every vertex $u \in V(G)$, where $a, b : V(G) \to \{0, 1, 2, \ldots\}$ are functions also given as part of the input. Decide if BOUNDED-DEGREE-SUBGRAPH is also polynomial time solvable, or NP-complete.