## NMAG403 - Combinatorics

## October 20, 2023 - Matching

## In class problems

14. Prove Hall's theorem (the difficult implication) from Tutte's theorem .
15. How many different perfect matchings does the Petersen graph have?
16. How many different perfect matchings does the complete graph $K_{n}$ have? How many of them contain a given fixed edge?
17. How many different spanning trees does the complete graph $K_{n}$ have? How many of them contain a given fixed edge?
18. Prove that every maximal matching in a graph $G$ has at least $\frac{\mu(G)}{2}$ edges.
19. Construct a 3 -regular graph with no perfect matching.
20. For every $k$, construct a graph of minimum degree at least $k$ with exactly one perfect matching.
21. Prove that if a graph $G$ contains no perfect matching, then it has a vertex $v$ such that each edge incident with $v$ belongs to a maximum matching of $G$.
