## NMAG403 - Combinatorics

October 20, 2023 – Matching

## In class problems

- 14. Prove Hall's theorem (the difficult implication) from Tutte's theorem .
- 15. How many different perfect matchings does the Petersen graph have?
- 16. How many different perfect matchings does the complete graph  $K_n$  have? How many of them contain a given fixed edge?
- 17. How many different spanning trees does the complete graph  $K_n$  have? How many of them contain a given fixed edge?
- 18. Prove that every maximal matching in a graph G has at least  $\frac{\mu(G)}{2}$  edges.
- 19. Construct a 3-regular graph with no perfect matching.
- 20. For every k, construct a graph of minimum degree at least k with exactly one perfect matching.
- 21. Prove that if a graph G contains no perfect matching, then it has a vertex v such that each edge incident with v belongs to a maximum matching of G.