Algorithmic Game Theory and Poker

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Sources

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- Moravcik, Matej, Evaluating public state space abstractions in extensive form games with an application in poker, 2014. https://is.cuni.cz/webapps/zzp/download/120166491/?lang=cs
- Publications of University of Alberta Computer Poker Research Group http://poker.cs.ualberta.ca/publications.html
- An Introduction to Counterfactual Regret Minimization http://modelai.gettysburg.edu/2013/cfr/

Class Overview

What we will NOT teach:

- How to actually win the money in poker.
- But, If you are already good poker player, some game theory will be helpful.

You will learn

- Some basic of the game theory.
- How to solve these games
- How it can be applied to the poker.
- Secrets behind top current computer poker players.

Syllabus

- Formal models of games.
- Nash equilibrium.
- Computational complexity, PPAD, NP, polynomial cases.
- Algorithms for solving different classes of games.
- Formalization of card games in game theory.
- Regret minimization.
- Counterfactual regret minimization - algorithm for solving large games with imperfect information (Poker).
- Game abstraction - how to make games reasonably small.
- Recent techniques used in the computer poker.
- Challanges and open problems.

About Us

- We are small group of people focusing on the game theory problems.
- We are also working on creating computer agents that will score well on Annual Computer Poker Competition (ACPC), and eventually beat world top human players in thefuture.

ACPC Results

- This year's ACPC results:

ANNUAL Computer Poker Competition

- Full results: `http://www.computerpokercompetition.org/`

Why is the Game Theory Interesting

It is young, rapidly developing field

- There are any new, theoretical and applied, research directions.
- There is lot of room for new discoveries.

It has been applied to many real world problems recently

- Airport security.
- Planing actions for the U.S. Coastal Guard.
- Protection of Wildlife in Uganda.
- Applications in network security.
- Applications in economy models.

For some interesing applications see Milind Tambe's webpage:
`http://teamcore.usc.edu/tambe/`

Why is the Poker Interesting

Unlike chess, it models several properties that are common for the real world problems:

- Imperfect information
- Stochastic events
- Quantification of winnings

There are also many other interesting properties of the Poker:

- Lot of strong human and computer players to play against.
- To complex to be solved just by brute force.
- Not "solved" like the chess.
- It is fun.

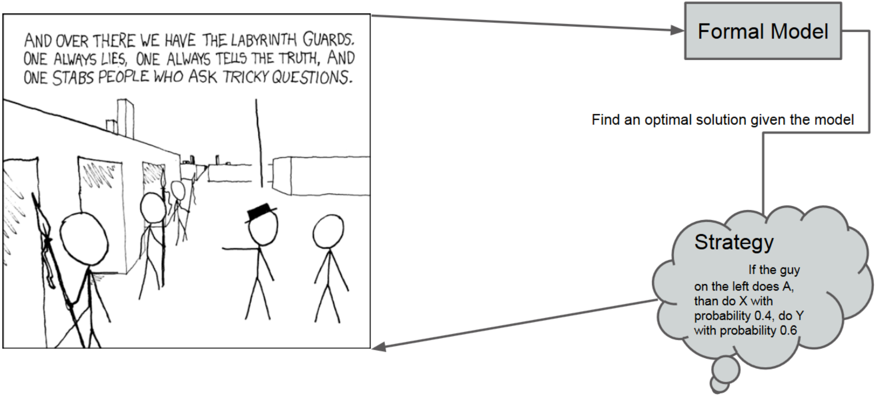
Usage of game theory in poker

Game theory is actually used by human players:

- Charts for the end of poker tournaments.
- Software for solving situations when the players have small amount of the chips ("SitNGo Wizard", "HoldemResources Calculator").
- Tools for modeling the game tree ("Equilab").
- For the better intuitive understanding of the game.

All of the current top computer poker players are based on results from the algorithmic game theory.

Game Theory Introduction



Game Theory Introduction

Game theory situation

- There are some agents/players involved in the situation
- The agents can take some action
- The outcome of the situation depends on the actions of the agents

Many properties

- Deterministic/random games
- Competitive/cooperative/coalition situation
- Simultaneous/sequential moves
- Finite/infinite games
- Perfect/imperfect information
- Repeated games
- ...

Game Theory Introduction

A game theory model typically defines

- The set of players
- The set of actions player may take
- Players outcome once the game is over

Let's have a look at the first formal model!

Normal Form Games

The normal form games is a model in which each player chooses his strategy, and then all players play simultaneously. The outcome depends on the actions chosen by the players.

Definition: Normal Form Game

is a tuple $\langle N, (A_i), (u_i) \rangle$, where

- N is the **finite** set of players
- A_i is the nonempty set of actions available to the player i
- u_i is a **payoff/utility** function for the player i. Let $A = \times_{i \in N} A_i$.
$$u_i : A \to \mathbb{R}$$

Example Games

Rock Paper Scissors

Popular game where two players simultaneously select either rock, paper or scissors. Player either wins, looses or draws.

Rock, paper, scissors, lizard, spock

Advanced version of the previous game.

Prisoner's dilemma

Two prisoners are being interrogated. Prisoner can either stay quiet or cooperate. If both stay quiet, they both get 2 years. If they both confess, they get 6 years. But if only one cooperates, he is offered a bargain and is freed, but the other prisoner gets 10 years

Game Theory Introduction

- If there are only two players ($|N| = 2$) , we can conveniently described the game using a table

	Rock	Paper	Scissors
Rock	(0, 0)	(-1, 1)	(1, -1)
Paper	(1, -1)	(0, 0)	(-1, 1)
Scissors	(-1, 1)	(1, -1)	(0, 0)

(a) Rock-Paper-Scissors

	Confess	Be quiet
Confess	(8, 8)	(0, 10)
Be quiet	(10, 0)	(2, 2)

(c) Prisoner's dilemma

- Rows/columns correspond to actions of player one/two
- In the cell (i, j), there are payoffs for both players - $u_1(i, j)$ and $u_2(i, j)$
- Which of the games above are constant sum games?

Normal Form Game Strategies

Definition: Pure Strategy

$a_i \in A_i$ is player i's pure strategy. This strategy is referred to as pure, because there's no probability involved. For example, the player can always play Scissors.

Definition: Mixed Strategy

is a probability measure over the player's pure strategies. The set of player i's mixed strategies is denoted as Σ_i. Given $\sigma_i \in \Sigma_i$, we denote the probability that the player chooses the action $a_j \in A_i$ as $\pi^{\sigma_i}(a_j)$ Mixed strategies allow a player to probabilistically choose actions. For example, his mixed strategy could be (Rock 0.4; Paper 0.4; Scissors 0.2)

Definition: Strategy profile

Is the set of all players' strategies (one for every player), denoted as $\sigma = (\sigma_0, \sigma_1 \dots \sigma_n)$. Finally, σ_{-i} refers to all the strategies in σ except σ_i.

Outcome

- Given a pure strategies of all players, we can easily compute the utilities. Player i's utility $= u_i(a)$
- How to compute the outcome if the players use mixed strategy (they randomize among the pure strategies)? We simply compute the expected value given the probability measure.
- Since the players choose the actions simultaneously, the events are independent and consequently $\pi^{\sigma}((a_0, a_1, \ldots, a_n)) = \pi^{\sigma_0}(a_0)\pi^{\sigma_1}(a_1)\ldots\pi^{\sigma_n}(a_n)$
- Using this fact, computing the expected value is easy

$$u_i(\sigma) = \sum_{a \in A} \pi^{\sigma}(a) u_i(a)$$

Best Response

- One of the key concepts, that you will see throughout the class
- Given the strategies $\sigma-i$ of the opponents, the **best response** is the strategy that maximizes the utility for the player.

Definition: Best Response

is a strategy σ_i^* such that $\forall \sigma_i' \in \Sigma_i$

$$u_i((\sigma_i^*, \sigma_{-i})) \geq u_i((\sigma_i', \sigma_{-i}))$$

- We denote the set of the best response strategies for the player i as the $BR_i(\sigma_{-i})$

Best Response

Lemma

For any best response strategy $\sigma_i \in BR_i(\sigma_{-i})$, all the actions that the player chooses with non-zero probability have the same expected value (given the $(sigma_{-i})$.

Lemma

The set best response set $BR_i(\sigma_{-i})$ is **convex**.

Dominant Strategies

- Some actions can be clearly poor choises, and it makes no sense for a rational player to take.
- Strategy σ_i^a **strictly dominates** σ_i^b iff for any σ_{-i}

$$u_i(\sigma_i^a, \sigma_{-i}) > u_i(\sigma_i^b, \sigma_{-i})$$

- Strategy σ_i^a **weakly dominates** σ_i^b iff for any σ_{-i}

$$u_i(\sigma_i^a, \sigma_{-i}) \geq u_i(\sigma_i^b, \sigma_{-i})$$

- Strategy is **strictly/weakly** dominated if there's a strategy that strictly/weakly dominates it.
- Strategies σ_i^a, σ_i^b are **intransitive** iff one neither dominates nor is dominated by the other.

Can a weakly/strictly dominated strategy be a best response?

Iterated elimination of dominated strategies

- A rational player does not play dominated strategy
- Iterated elimination of dominated strategies

	Left	Center	Right
Up	13, 3	1, 4	7, 3
Middle	4, 1	3, 3	6, 2
Down	-1, 9	2, 8	8, -1

- Let's iteratively remove the strategies that are dominated
- Can a weakly/strictly dominated strategy that we found during the iterated elimination be a best response in the original game?