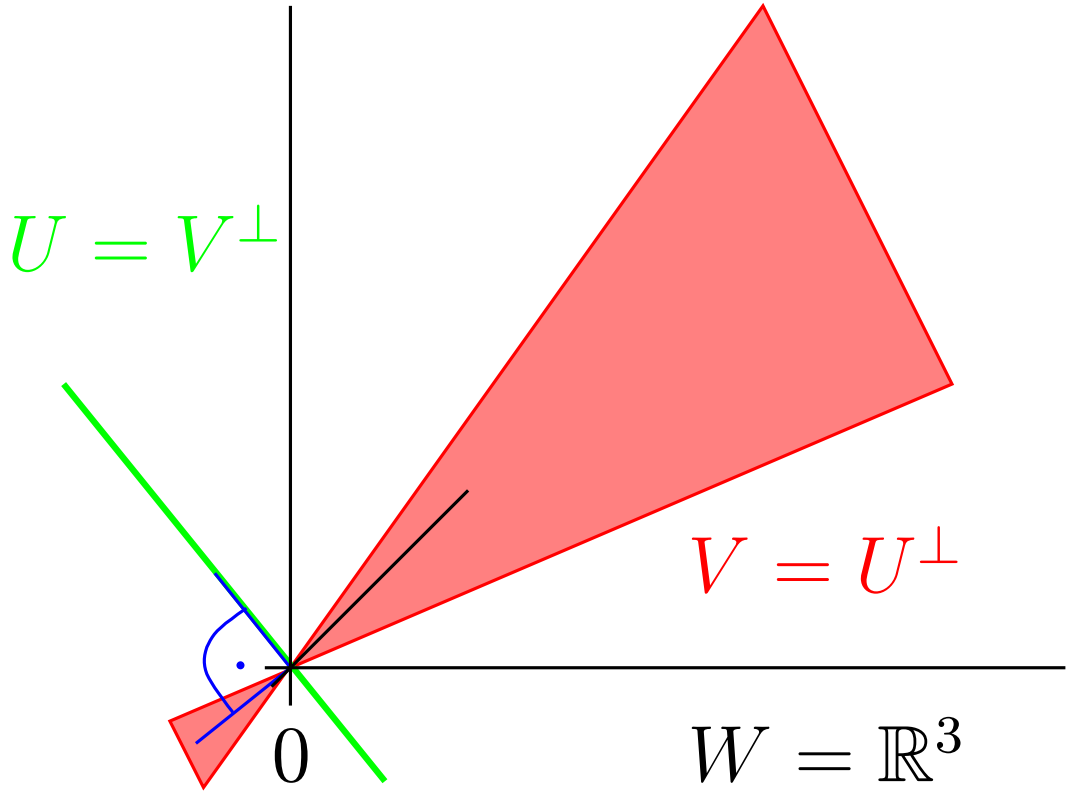
Ortogonální doplněk



Pro prostory určené maticí: $(\mathcal{R}(\mathbf{A}))^{\perp} = \mathrm{Ker}(\mathbf{A})$

Pro reálnou matici $\mathbf{A} = \begin{pmatrix} 1 & 3 & 4 & 5 \\ 2 & 6 & 3 & 0 \\ 3 & 9 & 15 & 9 \end{pmatrix} \sim \cdots \sim \begin{pmatrix} 1 & 3 & 4 & 5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Odtud $\mathrm{Ker}(\mathbf{A}) = \mathcal{L}\{(-13, 0, 2, 1)^T, (-3, 1, 0, 0)^T\}$.

Libovolné $\mathbf{x} \in \mathcal{R}(\mathbf{A})$ a libovolné $\mathbf{y} \in \mathrm{Ker}(\mathbf{A})$ splňují $\mathbf{x} \perp \mathbf{y}$, např.

$\mathbf{x} = \mathbf{u}^1 - 2\mathbf{u}^2 = (1, 3, 4, 5)^T - 2(0, 0, 1, -2)^T = (1, 3, 2, 9)^T$

$\mathbf{y} = \mathbf{v}^1 + 3\mathbf{v}^2 = (-13, 0, 2, 1)^T + 3(-3, 1, 0, 0)^T = (-22, 3, 2, 1)^T$

$\langle \mathbf{x} | \mathbf{y} \rangle = 1 \cdot (-22) + 3 \cdot 9 + 2 \cdot 2 + 9 \cdot 1 = 0$

$\langle \mathbf{x} | \mathbf{y} \rangle = \langle \mathbf{u}^1 - 2\mathbf{u}^2 | \mathbf{v}^1 + 3\mathbf{v}^2 \rangle = \langle \mathbf{u}^1 | \mathbf{v}^1 \rangle + 3\langle \mathbf{u}^1 | \mathbf{v}^2 \rangle - 2\langle \mathbf{u}^2 | \mathbf{v}^1 \rangle - 6\langle \mathbf{u}^2 | \mathbf{v}^2 \rangle = 0$