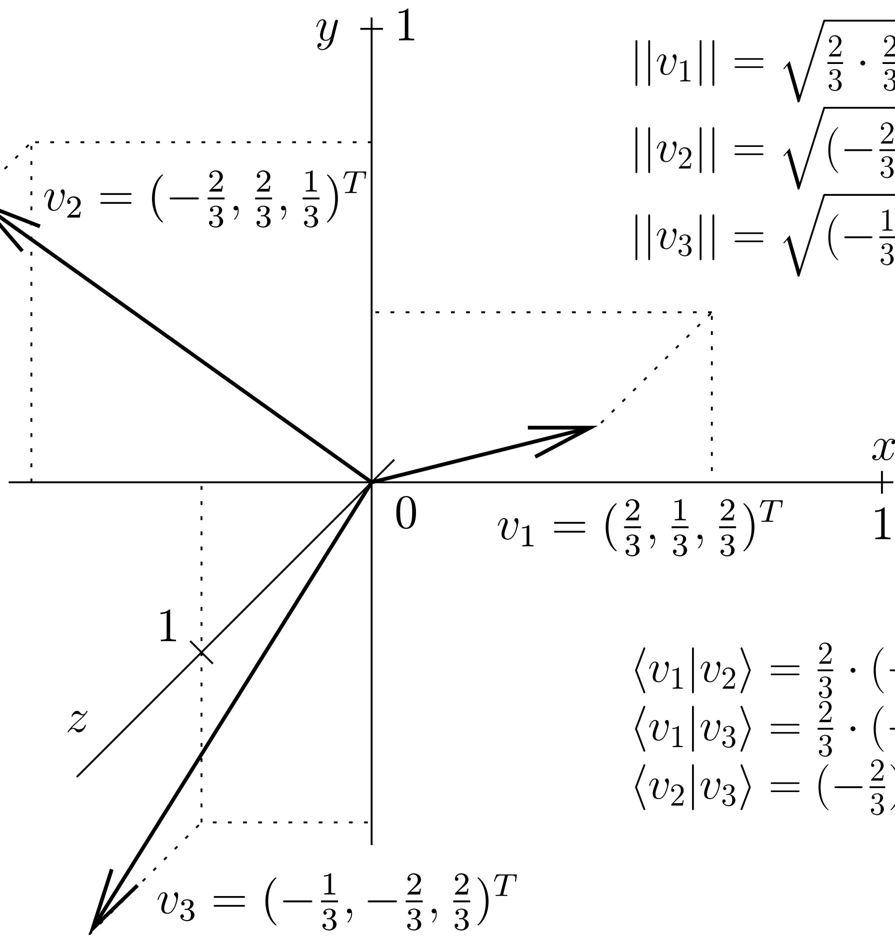
Příklad ortonormální báze v prostoru $\mathbb{R}^3$



$$||v_1|| = \sqrt{\frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{2}{3}} = 1$$

$$||v_2|| = \sqrt{(-\frac{2}{3}) \cdot (-\frac{2}{3}) + \frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3}} = 1$$

$$||v_3|| = \sqrt{(-\frac{1}{3}) \cdot (-\frac{1}{3}) + (-\frac{2}{3}) \cdot (-\frac{2}{3}) + \frac{2}{3} \cdot \frac{2}{3}} = 1$$

$$\langle v_1 | v_2 \rangle = \frac{2}{3} \cdot (-\frac{2}{3}) + \frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} = 0$$

$$\langle v_1 | v_3 \rangle = \frac{2}{3} \cdot (-\frac{1}{3}) + \frac{1}{3} \cdot (-\frac{2}{3}) + \frac{2}{3} \cdot \frac{2}{3} = 0$$

$$\langle v_2 | v_3 \rangle = (-\frac{2}{3}) \cdot (-\frac{1}{3}) + \frac{2}{3} \cdot (-\frac{2}{3}) + \frac{1}{3} \cdot \frac{2}{3} = 0$$

Vektor $u = (3,3,3)^T$ má vůči této bázi $Z = (v_1, v_2, v_3)$ souřadnice:

$$[u]_Z = (\langle u|v_1\rangle, \langle u|v_2\rangle, \langle u|v_3\rangle)^T = (5, 1, -1)^T$$

$\langle u|v_1\rangle = 3 \cdot \frac{2}{3} + 3 \cdot \frac{1}{3} + 3 \cdot \frac{2}{3} = 5$
$\langle u|v_2\rangle = 3 \cdot (-\frac{2}{3}) + 3 \cdot \frac{2}{3} + 3 \cdot \frac{1}{3} = 1$
$\langle u|v_3\rangle = 3 \cdot (-\frac{1}{3}) + 3 \cdot (-\frac{2}{3}) + 3 \cdot \frac{2}{3} = -1$

Zkouška:
$5 \cdot v_1 + 1 \cdot v_2 + (-1) \cdot v_3 =$
$= 5 \cdot (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})^T + 1 \cdot (-\frac{2}{3}, \frac{2}{3}, \frac{1}{3})^T + (-1) \cdot (-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})^T = (3,3,3)^T = u$