

# Matroid Theory Tutorials:

## (8) Gammoids

### Homework

**Definition 1.** Let  $G = (V, E)$  be a directed graph and  $X, Y \subseteq V$ . We say that  $X$  is connected to  $Y$ , if there are  $|Y|$  vertex disjoint paths from  $X$  to  $Y$ . (The paths are vertex disjoint not just internally vertex disjoint. We allow paths of length 0 if  $X \cap Y \neq \emptyset$ .)

**Definition 2.** Let  $G = (V, E)$  be a directed graph and  $S, T \subseteq V$ . A gammoid is a matroid over a set  $T$  and a subset  $X \subseteq T$  is independent if  $S$  is connected to  $X$  in  $G$ . A gammoid is strict if  $T = V$ .

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**HW 1.** Let  $\mathcal{G}$  be a gammoid over a set  $T$ . Show that rank of a set  $X \subseteq T$  is equal to the size of the minimum  $(S, X)$ -cut.

**HW 2.** Show that every uniform matroid is isomorphic to some gammoid. Is it possible to represent every uniform matroid as a strict gammoid?