## Matroid Theory Tutorials: (6) Representability – double series

## Homework

The following figure depicts a geometry representation of a matroid  $P_6$  (a geometry representation in the Eucledian plane is considered, i.e., an element set is independent if the corresponding point set is affine independent):

$$e_1 \quad e_2 \quad e_3$$

$$e_4 \quad e_5 \quad e_6$$
Matroid  $P_6$ .

**HW 1.** Show that for any representation of  $P_6$  in a form  $[I_3|D]$  (where the columns of D correspond the line  $\{e_4, e_5, e_6\}$ ) holds that all all elements of D are non-zero.

**HW 2.** Show that, if the matroid  $P_6$  is representable over a field  $\mathbb{F}$ , then there is a representation of  $P_6$  over  $\mathbb{F}$  in a form:

$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$
/1	0	0	1	1	1
0	1	0	1	a	b
$\sqrt{0}$	0	1	1	c	$_{d}$ /

Further, show that each representation in this form satisfies:

- 1. The column  $e_5$  does not contain two equal elements, i.e.,  $a \neq c \neq 1$  (the same holds for the column  $e_6$ ).
- 2.  $a \neq b$  and  $c \neq d$ .

**HW 3.** Show that  $P_6$  is not representable over GF(4). *Hint:* Use the previous exercises.

**HW 4.** Find a representation of  $P_6$  over  $\mathbb{F}_5$ .

Operation in GF(4).

+	0	1	x	x + 1
0	0	1	x	x+1
1	1	0	x + 1	<i>x</i>
x	x	x + 1	0	1
x + 1	x + 1	1  x	1	0
•	0	1	x	x + 1
0	0	0	0	0
1	0	1	x	x + 1
x	0	x	x + 1	1
x + 1	$1 \mid 0$	x + 1	1	x