Matroid Theory Tutorials: (2) Duals and Minors

Homework

HW 1. Let C be a circuit in matroid $\mathcal{M} = (E, \mathcal{I})$ and $e \in E$.

- 1. Show that if $e \in C$, then e is a loop in M or $C \setminus \{e\}$ is a circuit in M / e.
- 2. Show that it does not hold that if $e \notin C$, then C is a union of circuits in M / e.
- 3. Could C be a union of at least three circuits in M / e?

HW 2. Find dual matroids for the following matroids: $M(K_4), M(K_{2,3})$ and a matroid represented by the following matrix over \mathbb{Z}_2 :

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Other exercises

Exercise 1. Let G be a connected planar graph such that its spanning tree has k edges. What is the size of the spanning tree of G^* ? How it is related to the Euler's formula?