Matroid Theory Tutorials:
(2) Duals and Minors

Homework

HW 1. Let C be a circuit in matroid $\mathcal{M} = (E, \mathcal{I})$ and $e \in E$.

1. Show that if $e \in C$, then e is a loop in M or $C \setminus \{e\}$ is a circuit in M / e.
2. Show that it does not hold that if $e \notin C$, then C is a union of circuits in M / e.
3. Could C be a union of at least three circuits in M / e?

HW 2. Find dual matroids for the following matroids: $M(K_4), M(K_{2,3})$ and a matroid represented by the following matrix over $\mathbb{Z}_2$:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Other exercises

Exercise 1. Let G be a connected planar graph such that its spanning tree has k edges. What is the size of the spanning tree of G^*? How it is related to the Euler's formula?

Solution: *Rank of $M(G)$ is k. Thus, rank of $M^*(G) = m - k$, where m is a number of edges of G (and G^*). Since $M^*(G) = M(G^*)$, the spanning tree of G^* has $m - k$ edges.*

The Euler's formula states that for a connected planar graph $G = (V, E)$ holds that $n - m + s = 2$, where $n = |V|, m = |E|$ and s is a number of faces of G. Since $n = k + 1$ and s is a number of vertices of G^* (thus $s = m - k + 1$) we can proof the Euler's formula by properties of $M(G)$ and $M(G^*)$:

$$n - m + s = k + 1 - m + m - k + 1 = 2$$