Induced Ramsey-type results and binary predicates for point sets

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Charles University and
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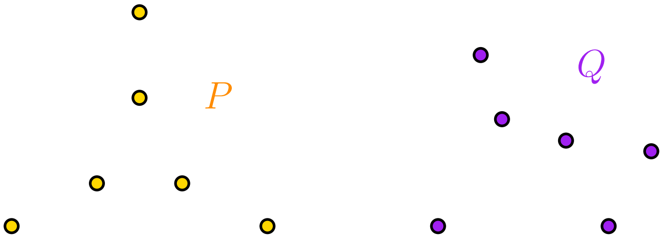

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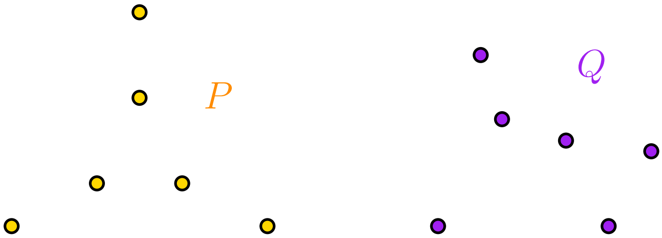

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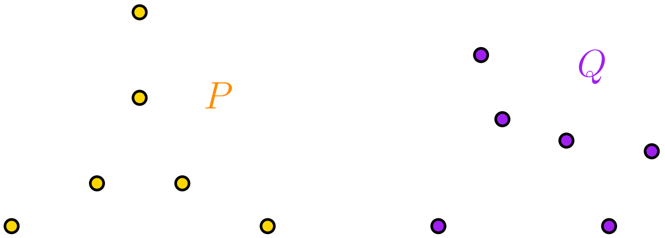

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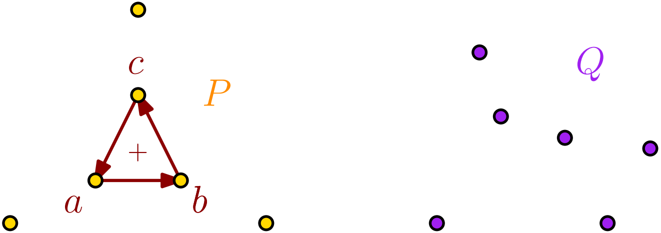

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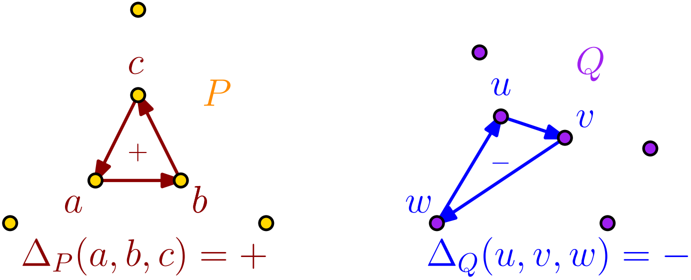

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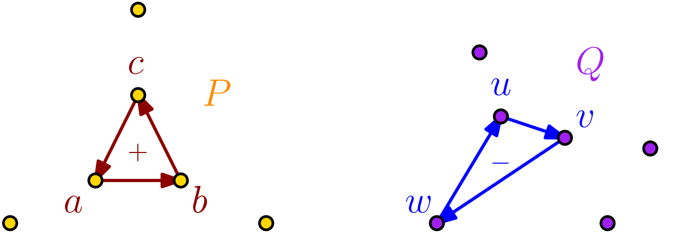

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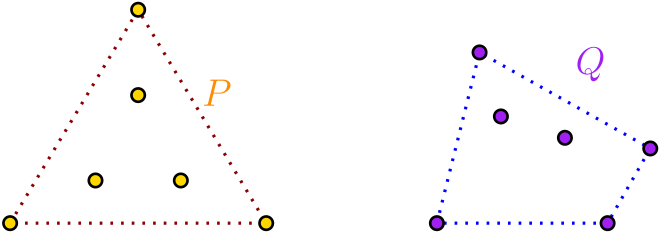

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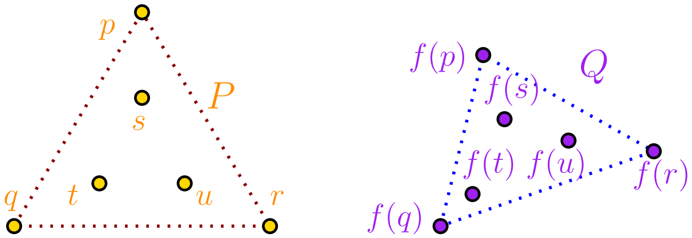

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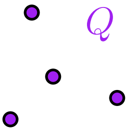

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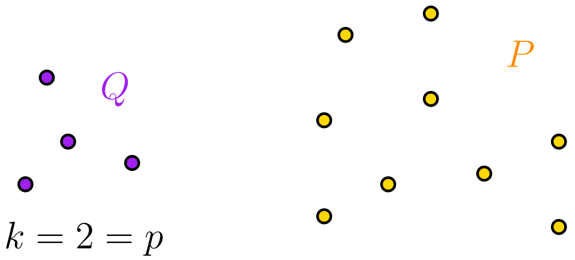

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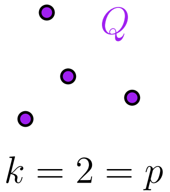

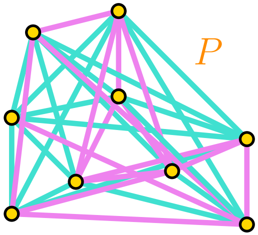

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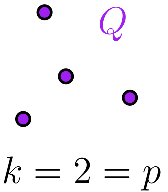

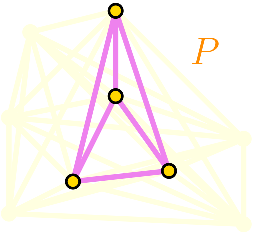

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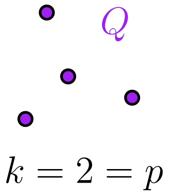

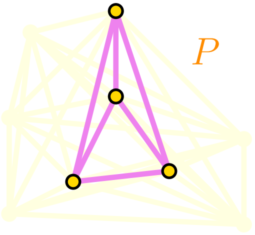

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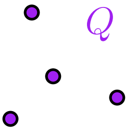

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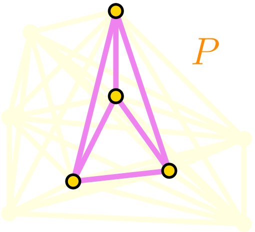

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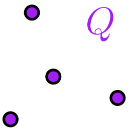

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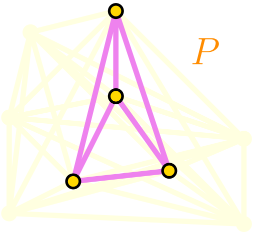

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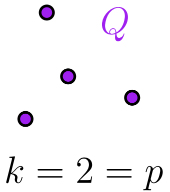

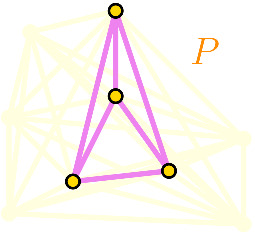

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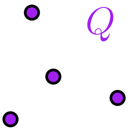

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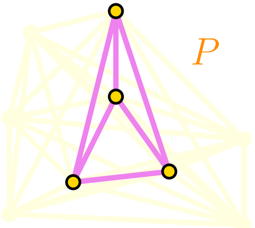

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  - For $k \in \mathbb{N}$, the non-convex 4-tuple is $(k, 2)$-Ramsey.

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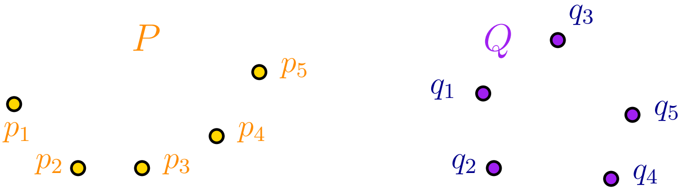

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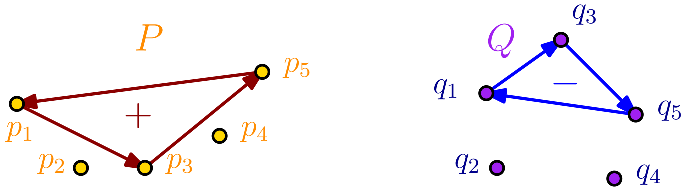

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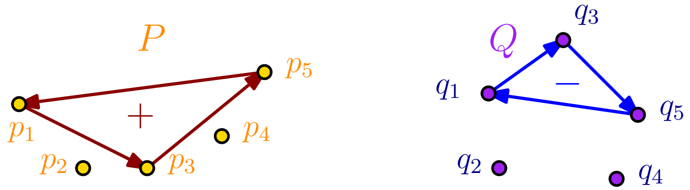

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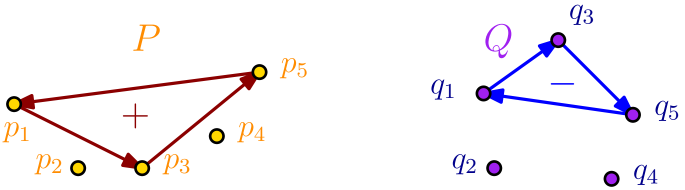

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- If a point set is ordered (k, p)-Ramsey, then it is (k, p)-Ramsey.

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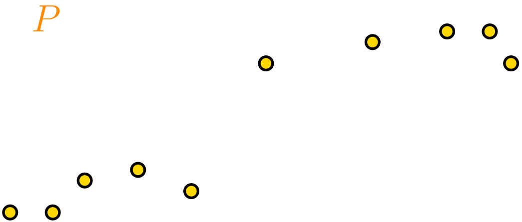

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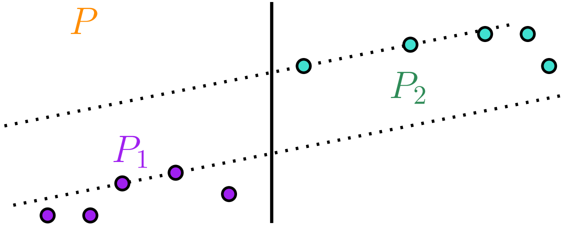

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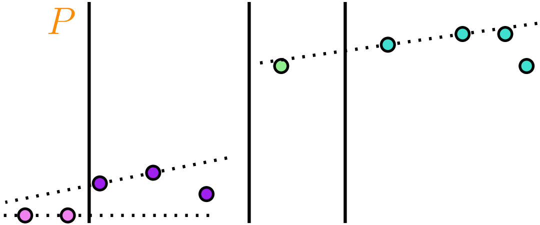

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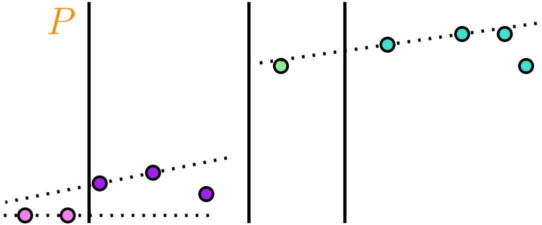

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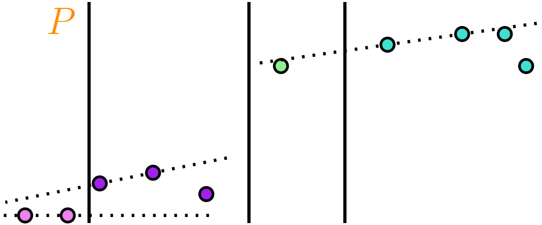

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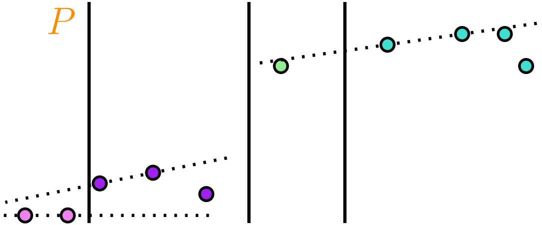

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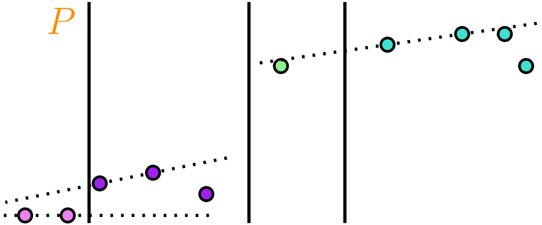

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- Theorem 1 has an application in the theory of combinatorial encodings of point sets.

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- We say that Γ encodes the order types if whenever there is a bijection $f : P \to Q$ such that $\Gamma_P(p_1, \dots, p_t) = \Gamma_Q(f(p_1), \dots, f(p_t))$ for every $(p_1, \dots, p_t) \in (P)_t$, then P and Q have the same order type via f.

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- For $t \in \mathbb{N}$ and a finite set Z, a t-ary point-set predicate with codomain Z is a collection $\Gamma = \{\Gamma_P \colon P \in \mathcal{P}\}$, where $\Gamma_P \colon (P)_t \to Z$.
- Example: ternary predicate $\Delta = \{\Delta_P \colon P \in \mathcal{P}\}$ with codomain $\{-, +\}$.
- We say that Γ encodes the order types if whenever there is a bijection $f \colon P \to Q$ such that $\Gamma_P(p_1, \ldots, p_t) = \Gamma_Q(f(p_1), \ldots, f(p_t))$ for every $(p_1, \ldots, p_t) \in (P)_t$, then P and Q have the same order type via f.
- For $n \in \mathbb{N}$, there are $2^{\Theta(n^3)}$ ternary functions $f \colon ([n])_3 \to \{-, +\}$, but only $2^{\Theta(n \log n)}$ order types of point sets of size n.

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- Is the encoding by Δ effective? Is it possible to use a binary predicate?

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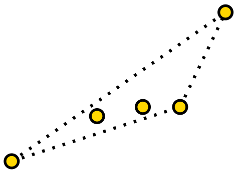

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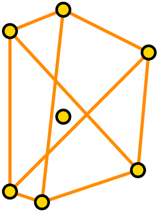

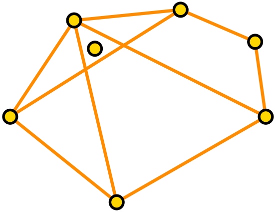

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Question 1

What is the growth rate of $h(k)$?

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