Isomorphisms of maps in linear time and recognition of symmetries of cyclic strings

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We show that one can test the map isomorphism between two maps of bounded genus in linear time.
A map is a 2-cell decomposition of a closed compact surface, i.e., an embedding of a graph such that every face is homeomorphic to an open disc. An automorphism of a map can be thought of as a permutation of the vertices which preserves the vertex-edge-face incidences in the embedding. It can be easily seen that to decide whether two maps are isomorphic can be done in quadratic time. While it is conjectured that there is no "truly subquadratic" algorithm for testing map isomorphism for unconstrained genus, we present a linear-time algorithm for computing the generators of the automorphism group of a map, parametrized by the genus of the underlying surface. The algorithm applies a sequence of local reductions and produces a labeled locally uniform map, while preserving the automorphism group. A map is locally uniform if for each vertex the associated cyclic sequence is the same. A set of generators of the automorphism group of the original map can be reconstructed from the automorphism group of the uniform map in linear time. We also extend the algorithm to non-orientable surfaces by making use of the antipodal double-cover. While the reduction part is the same for surfaces of negative and non-negative Euler characteristic $\chi(S)$, handling of the irreducible maps is different. If $\chi(S) < 0$, then there are just finitely many irreducible maps. Thus one can use brute force to decide the isomorphism (automorphism) of two input maps. In case of the sphere, torus, projective plane and Klein bottle there are infinitely many irreducible maps, and special algorithms are needed to resolve the isomorphism problem for labeled maps in linear time. In particular, for the sphere the irreducible maps includes the Platonic maps (5), the Archimedean maps (13), pseudo-rhombicubotahedron, and five infinite families, namely, prisms, antiprisms, cycles, dipoles and bouquets of circuits. In particular, for the sphere the computation of generators employs two sophisticated algorithms by Hopcroft and Wong 1974, and by Manacher 1975. The first algorithm, by Hopcroft and Wong, computes a cyclic symmetry of cyclic words of length n over a finite alphabet. The second algorithm, by Manacher, finds a maximal palindrome in a cyclic word over a finite alphabet. We shall discuss 2-dimensional analogues of these problems and their relation to symmetries of maps on the torus.
Maps on surfaces are excellent objects demostrating the universality of mathematics (and of informatics). They can be viewed from the point of view of topology, geometry and algebra. This is well demostrated by Mani theorem stating that the group automorphisms of a 3-connected planar graph can be realised as the automorphism group of an associated polyhedron (that means it is spherical group).

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