Algorithmic game theory — Homework 2[1]
Nash equilibria

assigned 4.11.2021, deadline 18.11.2021

If you wish to see your score on the web page, please choose a nickname and write it on the paper with your solutions (as well as your name), or send it by e-mail. Without the nickname I will not make your score public.

Homework 1. *Show that the following linear programs from the proof of the Minimax Theorem are dual to each other.*

(a) *For a matrix $M \in \mathbb{R}^{m \times n}$,* **[2]**

	Program P	Program D
Variables	$y_1, \ldots, y_n$	x_0
Objective function	$\min x^\top My$	$\max x_0$
Constraints	$\sum_{j=1}^n y_j = 1$,	$\mathbf{1}x_0 \leq M^\top x$.
	$y_1, \ldots, y_n \geq 0$.	

(b) *For a matrix $M \in \mathbb{R}^{m \times n}$,* **[2]**

	Program P'	Program D'
Variables	$y_0, y_1, \ldots, y_n$	$x_0, x_1, \ldots, x_m$
Objective function	$\min y_0$	$\max x_0$
Constraints	$\mathbf{1}y_0 - My \geq \mathbf{0}$,	$\mathbf{1}x_0 - M^\top x \leq \mathbf{0}$,
	$\sum_{j=1}^n y_j = 1$,	$\sum_{i=1}^m x_i = 1$,
	$y_1, \ldots, y_n \geq 0$.	$x_1, \ldots, x_m \geq 0$.

You may use the recipe for making dual programs from the lecture.

Homework 2. *Use the Lemke–Howson algorithm and compute a Nash equilibrium of the following bimatrix game* **[2]**

$$M = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 2 \\ 2 & 1 & 1 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 0 & 3. \end{pmatrix}$$

Start the computation by choosing the label 1.

Homework 3. *[Sperner's Lemma] Let S be a given subdivision of a triangle T in the plane. A* legal coloring *the vertices of S assigns one of three colors (red, blue, and green) to each vertex of S such that all the three colors are used on the vertices of T. Moreover, a vertex of S lying on an edge of T must have one of the two colors of the endpoints of this edge.*

Prove that, in every legal coloring of S, there is is a triangular face of S whose vertices are colored with all three colors.

Hint: Use a reduction to the END-OF-THE-LINE problem. **[3]**

[1] Information about the course can be found at http://kam.mff.cuni.cz/~balko/