Algorithmic game theory – Tutorial 3*

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1 ε-Nash and correlated equilibria

Let $G = (P, A, u)$ be a normal-form game of n players and let $\varepsilon > 0$. A strategy profile $s = (s_1, \dots, s_n)$ is an *ε-Nash equilibrium* if, for every player $i \in P$ and for every strategy $s'_i \in S_i$, we have $u_i(s_i; s_{-i}) \geq u_i(s'_i; s_{-i}) - \varepsilon$.

Let p be a probability distribution on A, that is, $p(a) \geq 0$ for every $a \in A$ and $\sum_{a \in A} p(a) = 1$. The distribution p is a *correlated equilibrium* in G if

$$\sum_{a_{-i} \in A_{-i}} u_i(a_i; a_{-i}) p(a_i; a_{-i}) \geq \sum_{a_{-i} \in A_{-i}} u_i(a'_i; a_{-i}) p(a_i; a_{-i})$$

for every player $i \in P$ and all pure strategies $a_i, a'_i \in A_i$.

Exercise 1. *Show that, in every normal-form game $G = (P, A, u)$, every convex combination of correlated equilibria is a correlated equilibrium.*

Exercise 2. *Let $G = (P = \{1, 2\}, A, u)$ be a normal-form game of two players with $A_1 = \{U, D\}$ and $A_2 = \{L, R\}$ with payoff function u depicted in Table 1.*

	L	R
U	(1,1)	(0,0)
D	$(1 + \frac{\varepsilon}{2}, 1)$	(500,500)

Table 1: A game from Exercise 2.

Show that there is an ε-Nash equilibrium s of G such that $u_i(s') > 10u_i(s)$ for every $i \in P$ and every Nash equilibrium s' of G. In other words, there might be games where some ε-Nash equilibria are far away from any Nash equilibrium.

Exercise 3. *Let $G = (P = \{1, 2\}, A, u)$ be a normal-form game of two players with $A_1 = \{U, D\}$ and $A_2 = \{L, R\}$ with payoff function u depicted in Table 2.*

	L	R
U	(6,6)	(2,7)
D	(7,2)	(0,0)

Table 2: A game from Exercise 3.

(a) *Compute all Nash equilibria of G and draw the convex hull of Nash equilibrium payoffs.*

(b) *Is there any correlated equilibrium of G (for some ditribution p) that yields payoffs outside this convex hull?*

*Information about the course can be found at http://kam.mff.cuni.cz/~balko/