Algorithmic game theory – Tutorial 5*

December 16, 2018

1 Revenue-maximizing auctions

Exercise 1. *Let F be the uniform probability distribution on $[0,1]$. Consider a single-item auction with two bidders 1 and 2 that have probability distributions $F_1 = F$ and $F_2 = F$ on their valuations. Prove that the expected revenue obtained by the Vickrey auction (with no reserve) is $1/3$.*

Exercise 2. *Compute the virtual valuation function of the following probability distributions and show which of these distributions are regular (meaning the virtual valuation function is strictly increasing).*

(a) The uniform distribution $F(z) = z/a$ on $[0,a]$ with $a > 0$,

(b) The exponential distribution $F(z) = 1 - e^{-\lambda z}$ with rate $\lambda > 0$ on $[0,\infty)$,

Exercise 3. *Consider a single-item auction where bidder i draws his valuation from his own regular distribution F_i, that is, the probability distributions $F_1,\dots,F_n$ can be different but all virtual valuation functions $\varphi_1,\dots,\varphi_n$ are strictly increasing.*

(a) Give a formula for the winner's payment in an optimal auction, in terms of the bidders' virtual valuation functions φ_i. Verify that if $F_1 = \cdots = F_n$ are uniform probability distributions on $[0,1]$, then your formula yields Vickrey auction with reserve price $1/2$.

(b) Find an example of an optimal auction in which the highest bidder does not win, even if he has a positive virtual valuation.

Hint: It suffices to consider two bidders with valuations from different uniform distributions.

Exercise 4. *Prove that the Knapsack auction allocation rule x^G induced by the greedy $(1/2)$-approximation algorithm covered in the lecture is monotone.*

*Information about the course can be found at http://kam.mff.cuni.cz/~balko/