Algorithmic game theory – Tutorial 4*

November 25, 2018

1 Regret minimization

Exercise 1. *Let A be the Polynomial weights algorithm with parameter $\eta \in (0, 1/2]$ and with external regret at most $\alpha/\eta + \beta\eta T$ for some constants α, β (that may depend on the number N of actions). We showed that choosing $\eta = \sqrt{\alpha/(T\beta)}$ minimizes the bound. Modify this algorithm so that we obtain an external regret bound that is at most $O(1)$-times larger that the original bound for any T. In particular, you cannot run A with a parameter η that depends on T.*

Hint: Partition the set $\{1, \dots, T\}$ into suitable intervals I_m for $m = 0, 1, 2, \dots$ and run A with a suitable parameter η_m in every step from I_m.

Exercise 2. *Consider the following setting in which the agent A tries to learn the setup in an adversary environment while using information given to him by a set S_0 of N experts. The setting proceeds in a sequence of steps $t = 1, \dots, T$. In every step t, the environment picks $y_t \in \{0, 1\}$, which is unknown to A and to the experts, and each expert i gives a recommendation $f_{i,t} \in \{0, 1\}$ to A. The agent A then makes prediction $z_t \in \{0, 1\}$ based on the experts' advice and then sees y_t. The goal of A is to minimize the number $M^T(A)$ of steps t in which $z_t \neq y_t$.*

(a) *Assume that, in each step t, the agent A selects z_t to be the majority vote of all experts from S_{t-1} and, after seeing y_t, he lets S_t be the number of agents $i \in S_{t-1}$ with the right guess $f_{i,t} = z_t$. Also assume that there is a perfect expert that always guesses right. Prove that then $M^T(A) \leq \log_2 N$.*

(b) *Modify the above algorithm of the agent so that $M^T(A) \leq O((m + 1) \log_2 N)$ when the best expert makes $m \geq 0$ mistakes.*

Exercise 3. *Show formally that every correlated equilibrium is a coarse correlated equilibrium.*

Exercise 4. *Show an example with $N = 3$ where the ratio between the external regret and swap regret can be unbounded.*

Clarification: you need to choose only a sequence of actions $a^1, \dots, a^T$, $a^i \in X = \{1, 2, 3\}$, and a loss sequence $\ell_a^1, \dots, \ell_a^T$ for every $a \in X$ (no adversary or algorithm).

Exercise 5. *Show that the swap regret is at most N times larger than the internal regret.*

*Information about the course can be found at http://kam.mff.cuni.cz/~balko/