Algorithmic game theory – Tutorial 3[*]

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1 ε-Nash and correlated equilibria

Exercise 1. *Show that, in every normal-form game $G = (P, A, u)$ of n players, for every Nash equilibrium there exists a corresponding correlated equilibrium. In particular, show that if $s^* = (s_1^*, \ldots, s_n^*)$ is a Nash equilibrium in G, then the product probability distribution p_{s^*} given by*

$$p_{s^*}(a) = \prod_{j=1}^{n} s_j^*(a_j),$$

for every $a = (a_1, \ldots, a_n) \in A$, is a correlated equilibrium in G.

Exercise 2. *Show that, in every normal-form game $G = (P, A, u)$, every convex combination of correlated equilibria is a correlated equilibrium.*

Exercise 3. *Let $G = (P = \{1, 2\}, A, u)$ be a normal-form game of two players with $A_1 = \{U, D\}$ and $A_2 = \{L, R\}$ with payoff function u depicted in Table 1.*

	L	R
U	(1,1)	(0,0)
D	$(1 + \frac{\varepsilon}{2}, 1)$	(500,500)

Table 1: A game from Exercise 3.

Show that there is an ε-Nash equilibrium s of G such that $u_i(s') > 10u_i(s)$ for every $i \in P$ and every Nash equilibrium s' of G. In other words, there might be games where some ε-Nash equilibria are far away from any Nash equilibrium.

Exercise 4. *Let $G = (P = \{1, 2\}, A, u)$ be a normal-form game of two players with $A_1 = \{U, D\}$ and $A_2 = \{L, R\}$ with payoff function u depicted in Table 2.*

	L	R
U	(6,6)	(2,7)
D	(7,2)	(0,0)

Table 2: A game from Exercise 4.

(a) *Compute all Nash equilibria of G and draw the convex hull of Nash equilibrium payoffs.*

(b) *Is there any correlated equilibrium of G (for some ditribution p) that yields payoffs outside this convex hull?*

Exercise 5. *Let $G = (P = \{1, 2\}, A, u)$ be a normal-form game of two players with $A_1 = \{U, D\}$ and $A_2 = \{L, R\}$ with payoff function u depicted in Table 3.*

	L	R
U	(4,4)	(1,5)
D	(5,1)	(0,0)

Table 3: A game from Exercise 5.

(a) *Compute all Nash equilibria of G and draw the convex hull of Nash equilibrium payoffs.*

(b) *Determine the set of all correlated equilibria of G.*