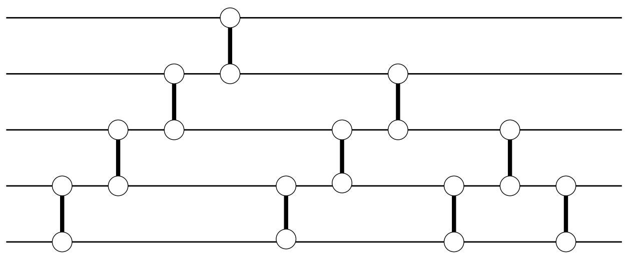
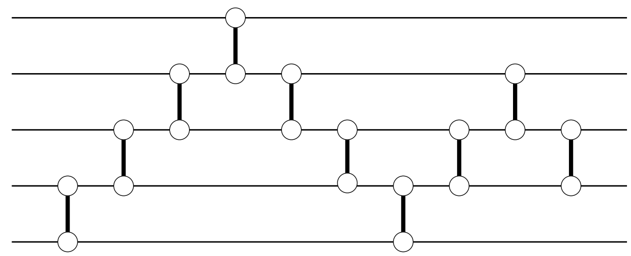
GENERALIZED PSEUDOTRIANGULATIONS

Michel Pocchiola, Paris/Prague, July 2010

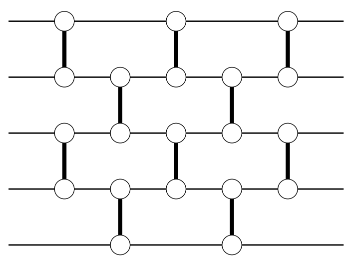
http://www.people.math.jussieu.fr/~pocchiola/



"bubblesort"



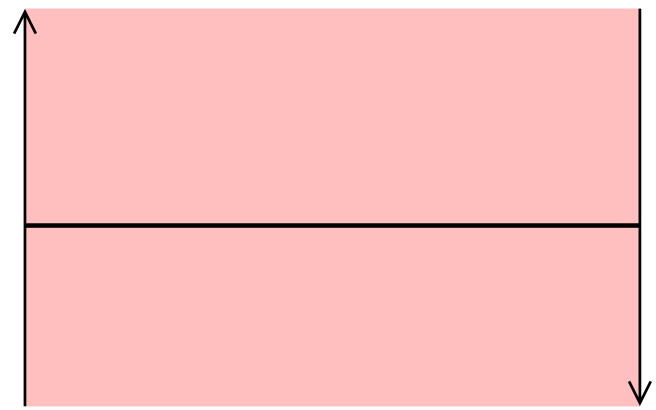
"cocktail-shaker sort"

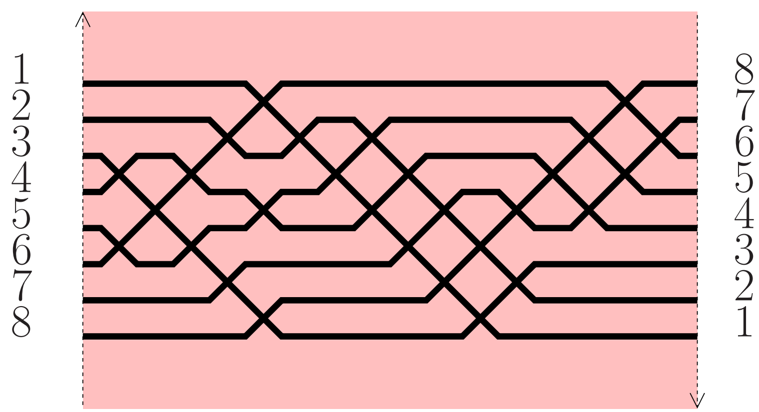


"odd-even transposition sort"

mainly based on joint work with Vincent Pilaud, PhD student.

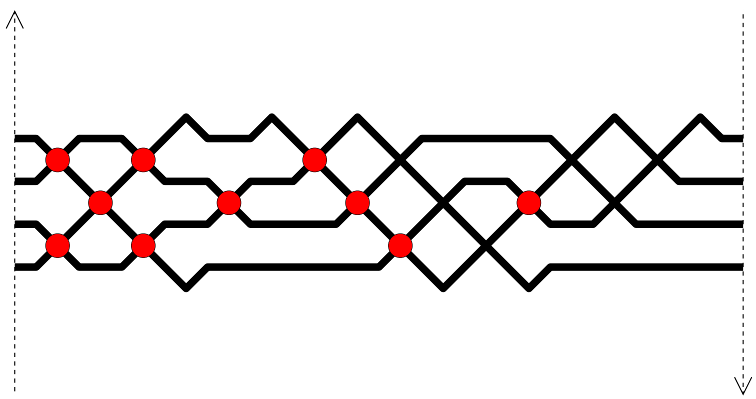
BACKGROUND

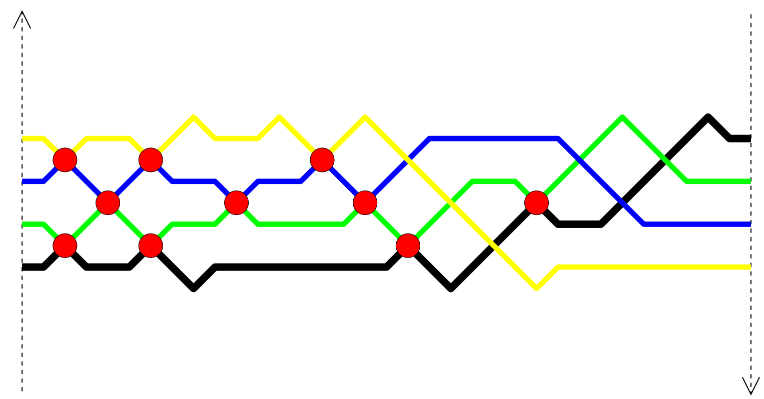




Möbius strip without boundary - pseudolines - arrangement of pseudolines

DEFINITIONS



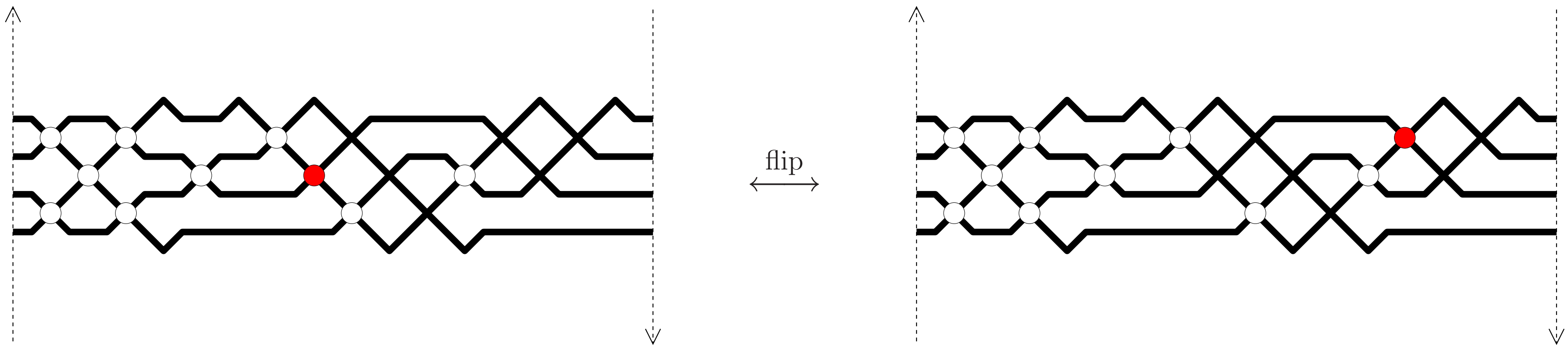


An arrangement of pseudolines with contact points is a finite family of pseudolines that intersect pairwise in a finite number of points of which only one is a transversal intersection point. We will restrict our attention to the case where the intersection of three pseudolines is empty.

A network is the support $\bigcup \Gamma$ of an arrangement of pseudolines with contact points Γ. Observe that an arrangement of pseudolines with contact points is entirely determinated by its support and its set of contact points.

The atoms of a network are the sets of contact points of the arrangements of pseudolines with contact points with support that network. The complex of atoms of a network is the set of subsets of the atoms of the network ordered by reverse inclusion and augmented with a minimum element.

COMPLEX OF ATOMS - FLIP GRAPH



TH 1. *The complex of atoms of a network is an abstract simple polytope.* □

Is it a topological ball?
Is it a matroid polytope?
Is it a polytope?

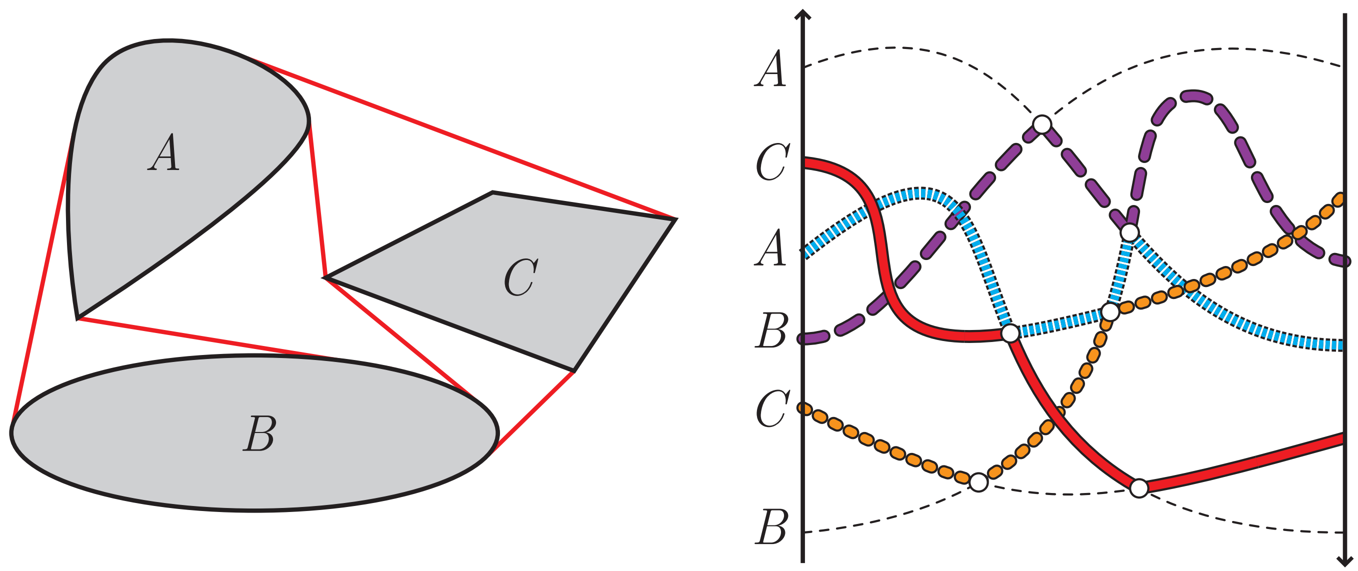
TWO PSEUDOLINES

The complex of atoms of the support of an arrangement of two pseudolines with m contact points is a simplex of dimension m.

THREE PSEUDOLINES

The complex of atoms of the support of an arrangement of three pseudolines with contact points is a simple polytope with three more facets than its dimension. Futhermore any simple polytope with three more facets than its dimension can be realized like this. (F. Santos 09).

PSEUDOTRIANGULATIONS

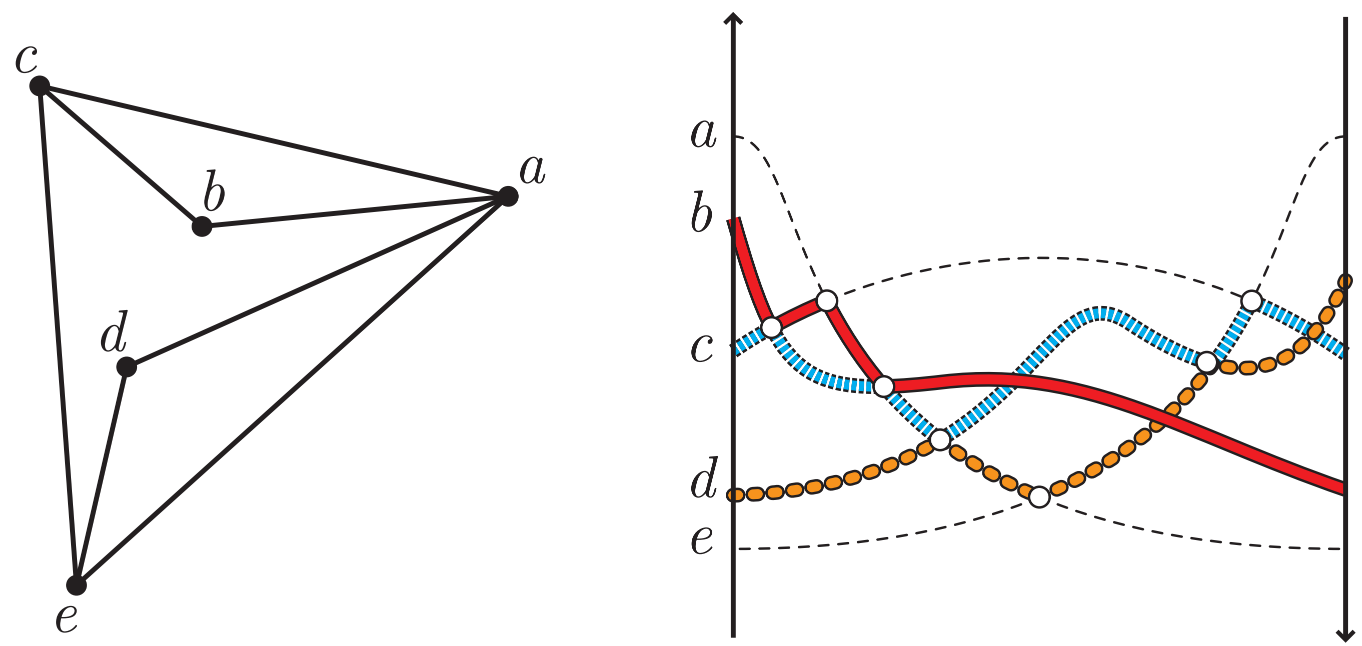


(a) (b)

TH 2. *Let Δ be a finite planar set of pairwise disjoint convex bodies in general position, let Δ^* be its dual arrangement, and let $\mathcal{N}(\Delta)$ be the closure of the support of Δ^* minus its first level. Then $\mathcal{N}(\Delta)$ is a network and its complex of atoms is isomorphic to the complex of pseudotriangulations of Δ.* □

P. & Vegter'96.

POINTED PSEUDOTRIANGULATIONS

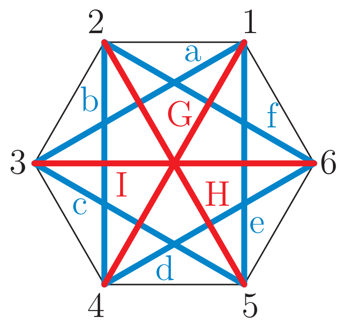


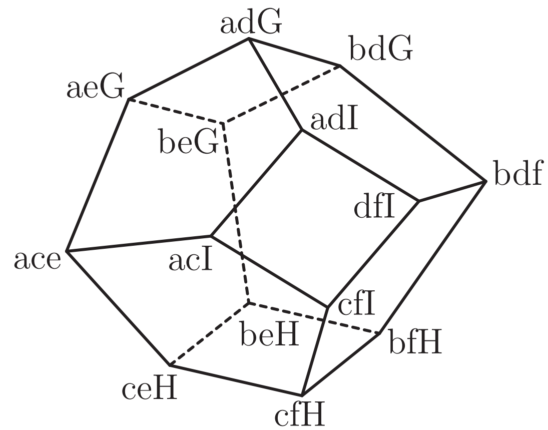
(a) (b)

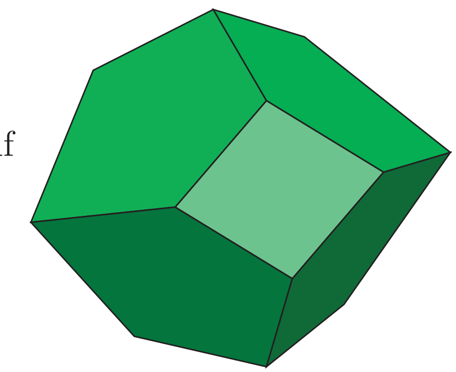
TH 3. *Let Δ be a finite planar set of points in general position, let Δ^* be its dual arrangement, and let $\mathcal{N}(\Delta)$ be the closure of the support of Δ^* minus its first level. Then $\mathcal{N}(\Delta)$ is a network and its complex of atoms is isomorphic to the complex of (pointed) pseudotriangulations of Δ.* □

The complex of pointed pseudotriangulations is polytopal .... if the term "planar" refers to the euclidean plane (Rote, Santos and Streinu'03).

TRIANGULATIONS OF THE n-GON



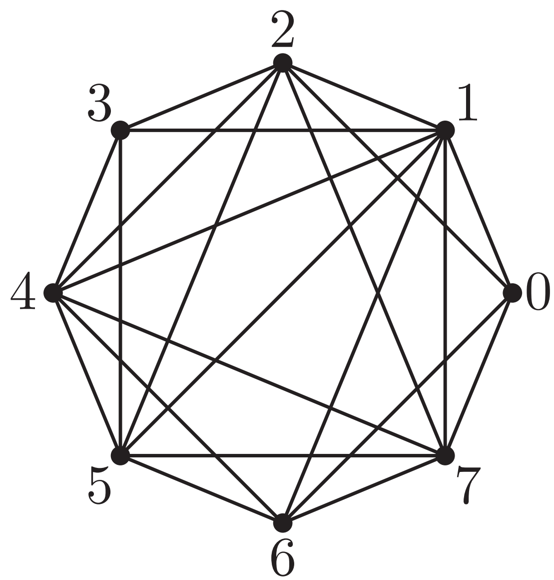




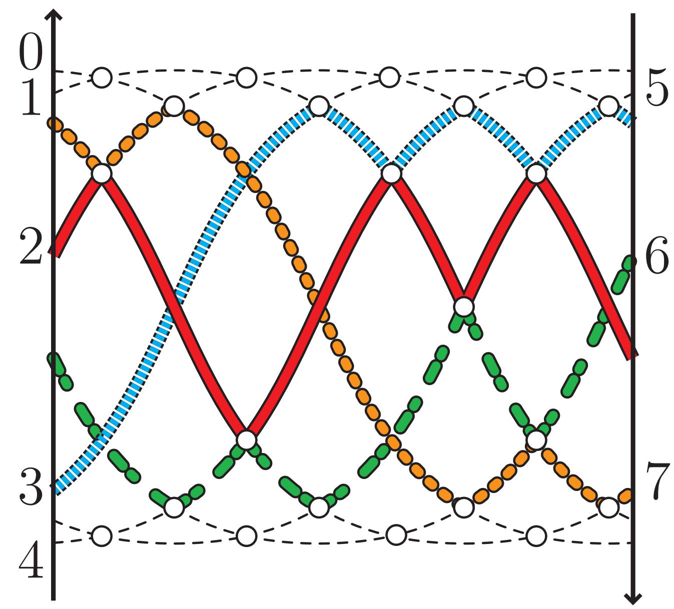
TH 4. *Let Δ be a finite planar set of n points in convex position and let $\mathcal{N}_1(\Delta)$ be the closure of the support of the dual arrangement of Δ minus its first level. Then $\mathcal{N}_1(\Delta)$ is a network and its complex of atoms is isomorphic to the complex of triangulations of Δ.* □

The complex of triangulations of Δ is a polytope : the so-called associahedron.

MULTITRIANGULATIONS



(a)



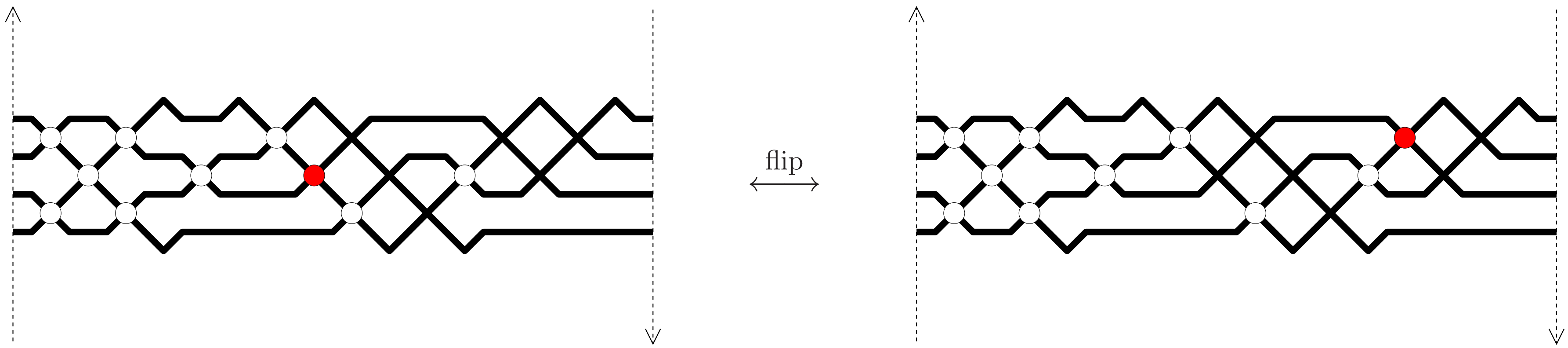
(b)

TH 5. *Let Δ be a finite planar set of n points in convex position, let $k \geq 1$ be a natural number, and let $\mathcal{N}_k(\Delta)$ be the closure of the support of the dual arrangement of Δ minus its k first levels. Then $\mathcal{N}_k(\Delta)$ is a network and its complex of atoms is isomorphic to the complex of k-triangulations of Δ.* □

Capeyleas & Pach'92, Jonsson'05, Pilaud and Santos'08, and others

The complex of k-triangulations of Δ is a topological sphere (Jonsson'05), and the complex of 2-triangulations of Δ_8 is a polytope (Bokowski and Pilaud'09).

COMPLEX OF ATOMS - FLIP GRAPH



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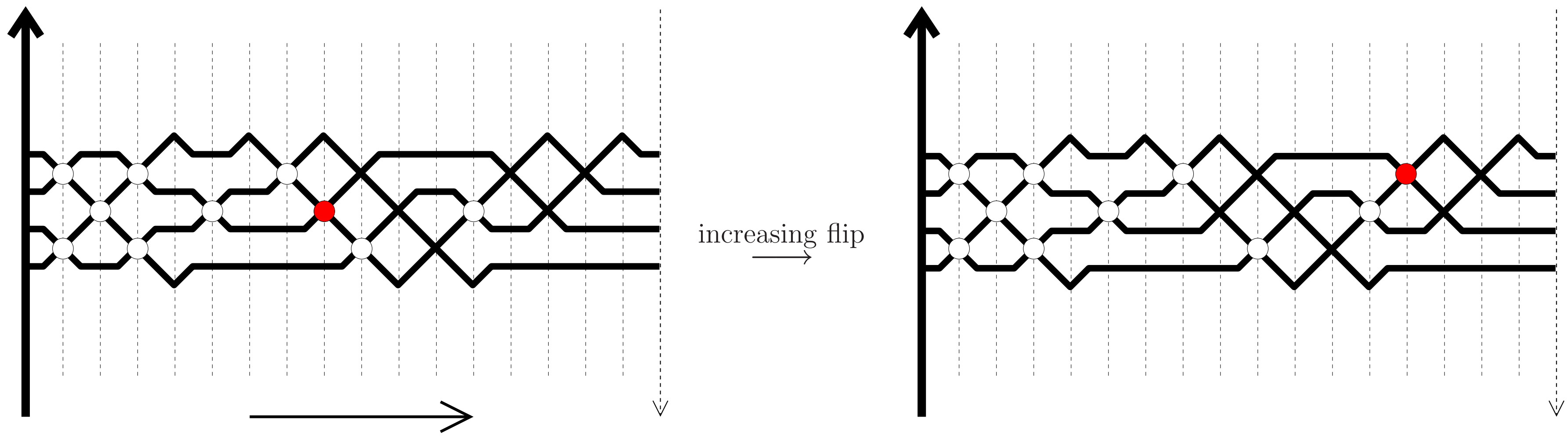
DIRECTED FLIP GRAPHS

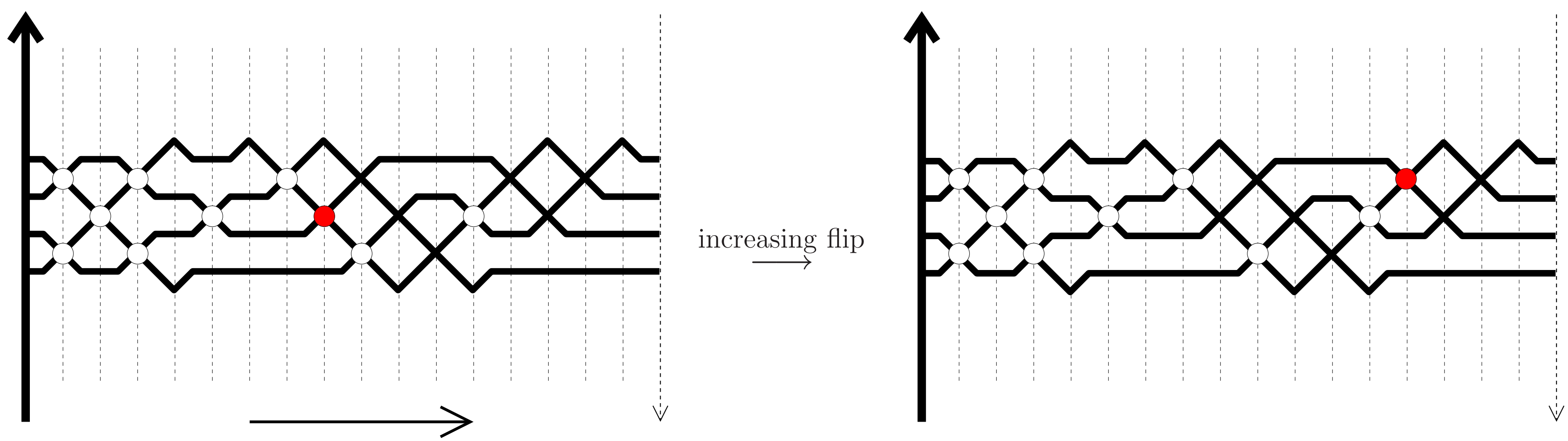
universal covering - partial order - filters and cuts



$$I \mapsto \mathcal{P}(I; \mathcal{N})$$

TH 6. *Let I be a filter of a network $\mathcal{N}$. Then the digraph $\mathcal{P}(I; \mathcal{N})$ is acyclic.* □

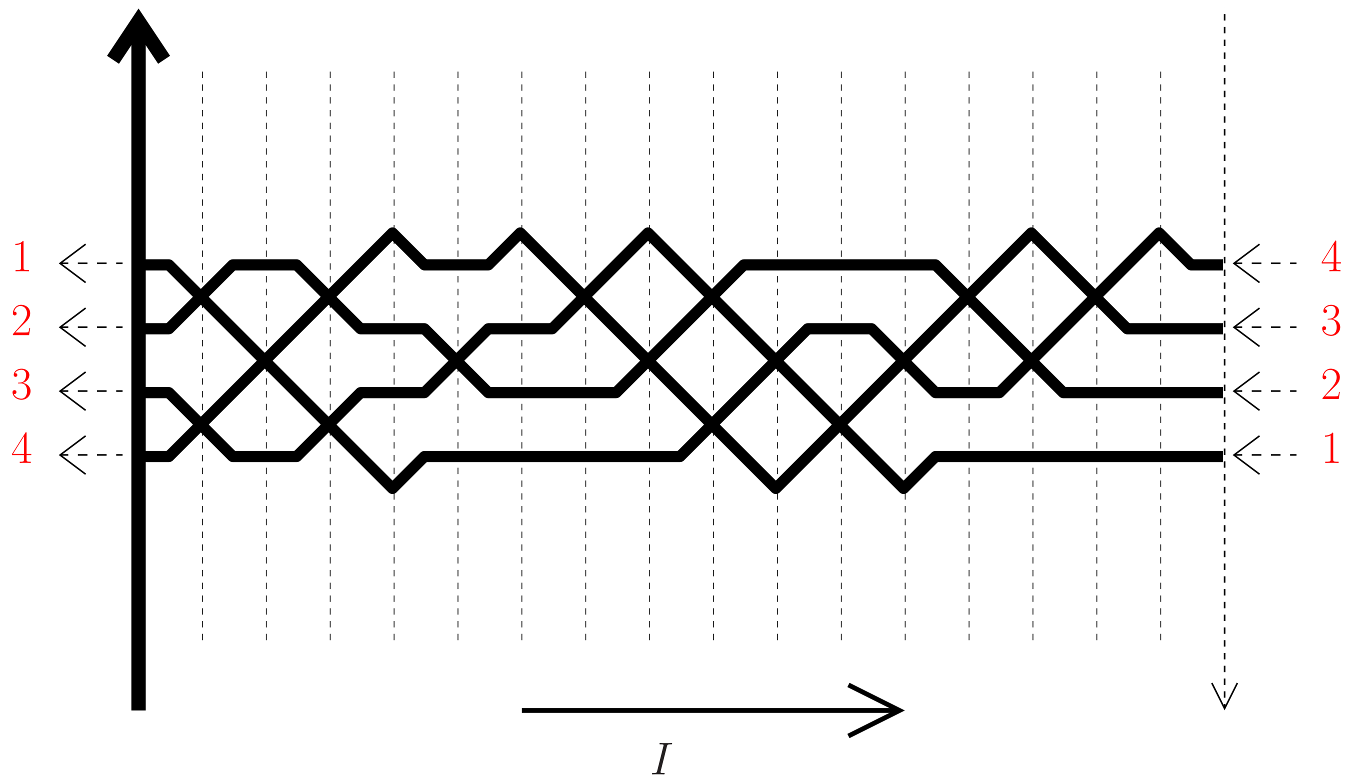
UNIQUE SOURCE AND UNIQUE SINK



TH 7. *Let I be a filter of a network $\mathcal{N}$. Then the digraph $\mathcal{P}(I;\mathcal{N})$ has a unique source $G(I)$ which is characterized by the property that for any pair of pseudolines γ and γ', for any contact point u between γ and γ' one has $u_I \preceq v_I$ where u_I is the lift of u in $I \setminus \iota(I)$ and v_I is the lift in $I \setminus \iota(I)$ of the crossing point v of γ and γ'.* □

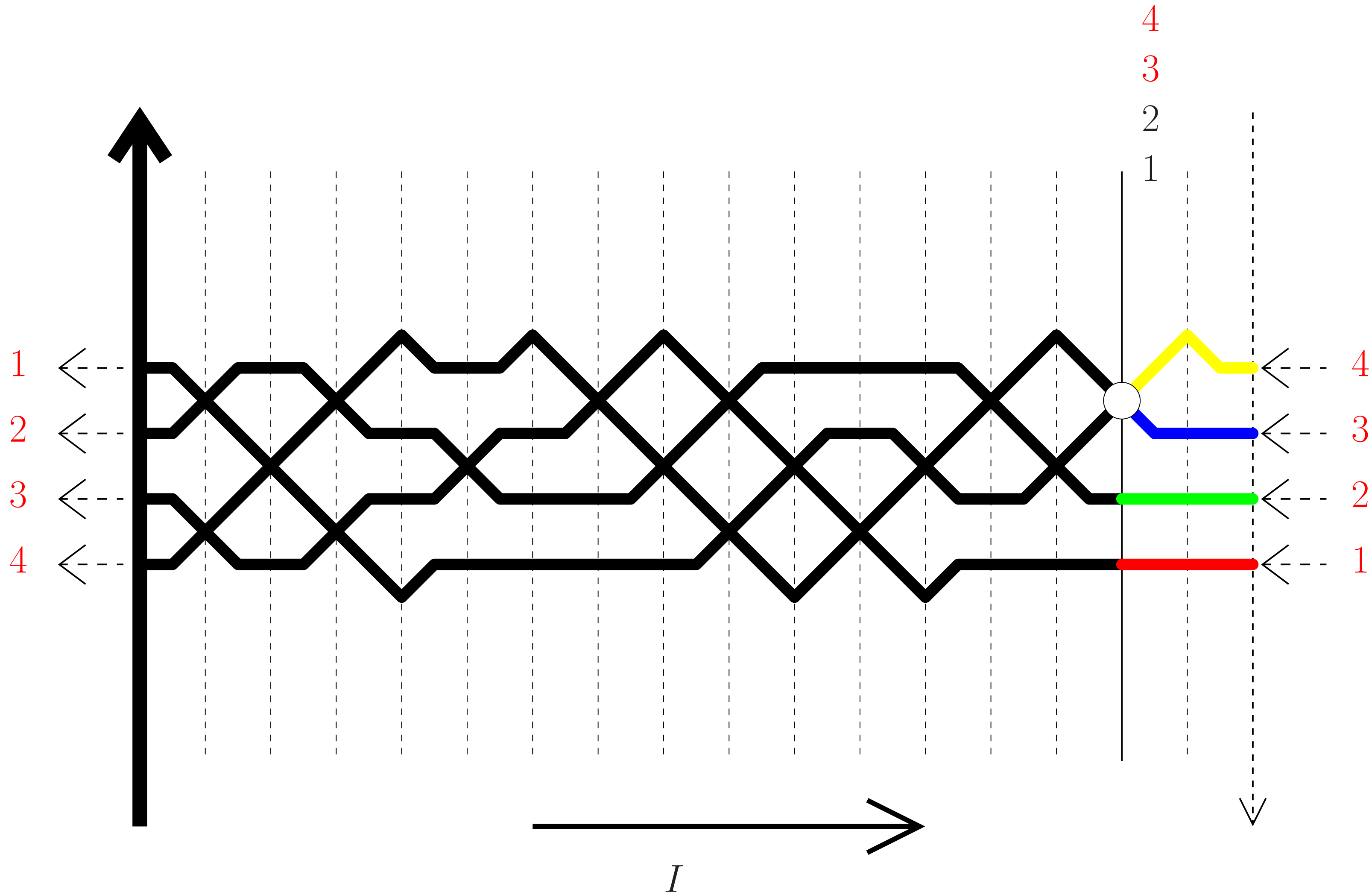
TH 8. *Let $\mathcal{N}$ be a network. Then the graph $\mathcal{P}(\mathcal{N})$ is connected.* □

COMPUTING THE SOURCE

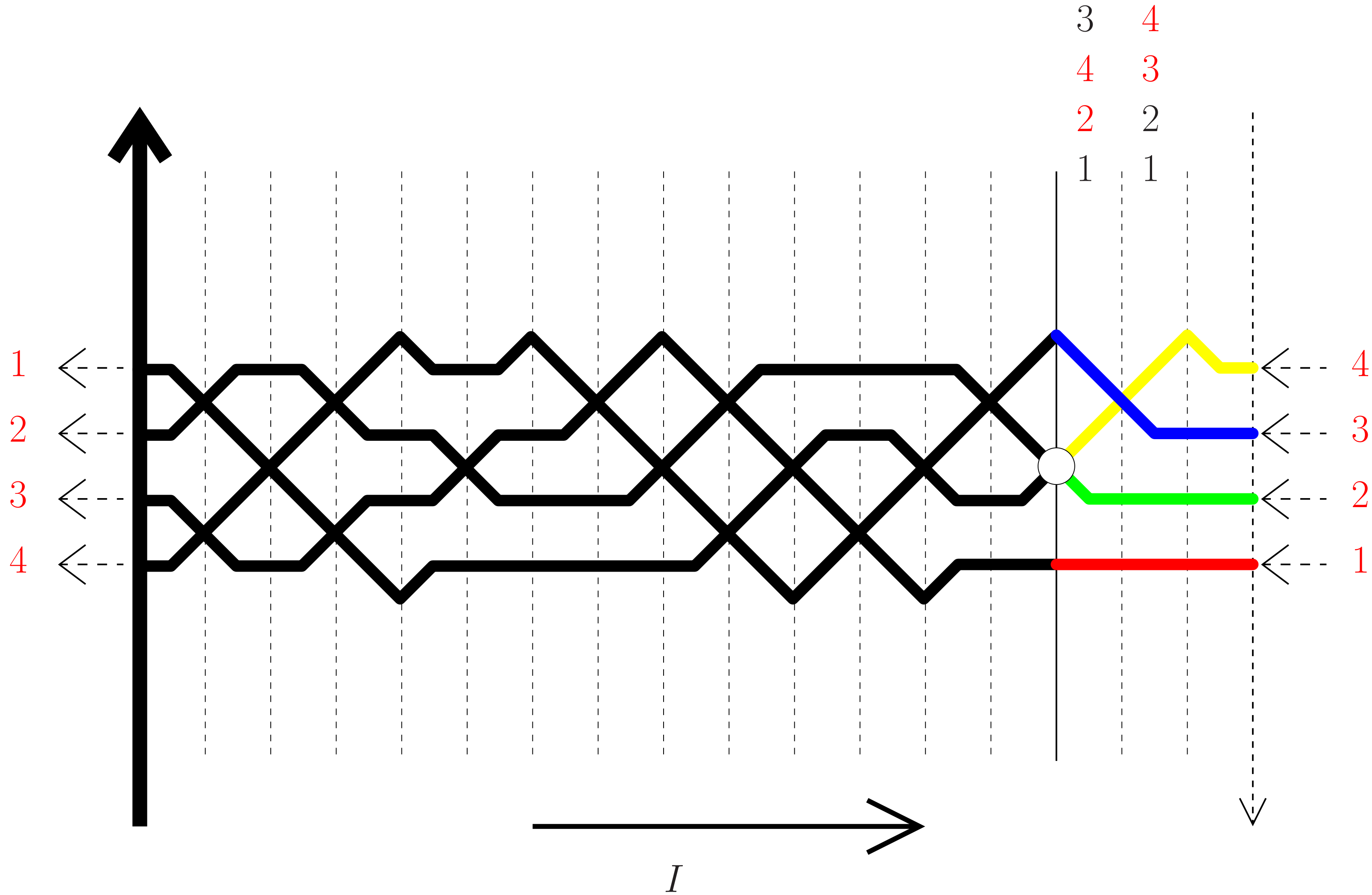


TH 9. *The digraph $\mathcal{P}(I;\mathcal{N})$ has a unique source $G(I)$ obtained as the result of sorting the permutation $\tau = (n, n-1, \ldots, 1)$ with the primitive sorting network $\mathcal{S}(I)$.* □

COMPUTING THE SOURCE



COMPUTING THE SOURCE



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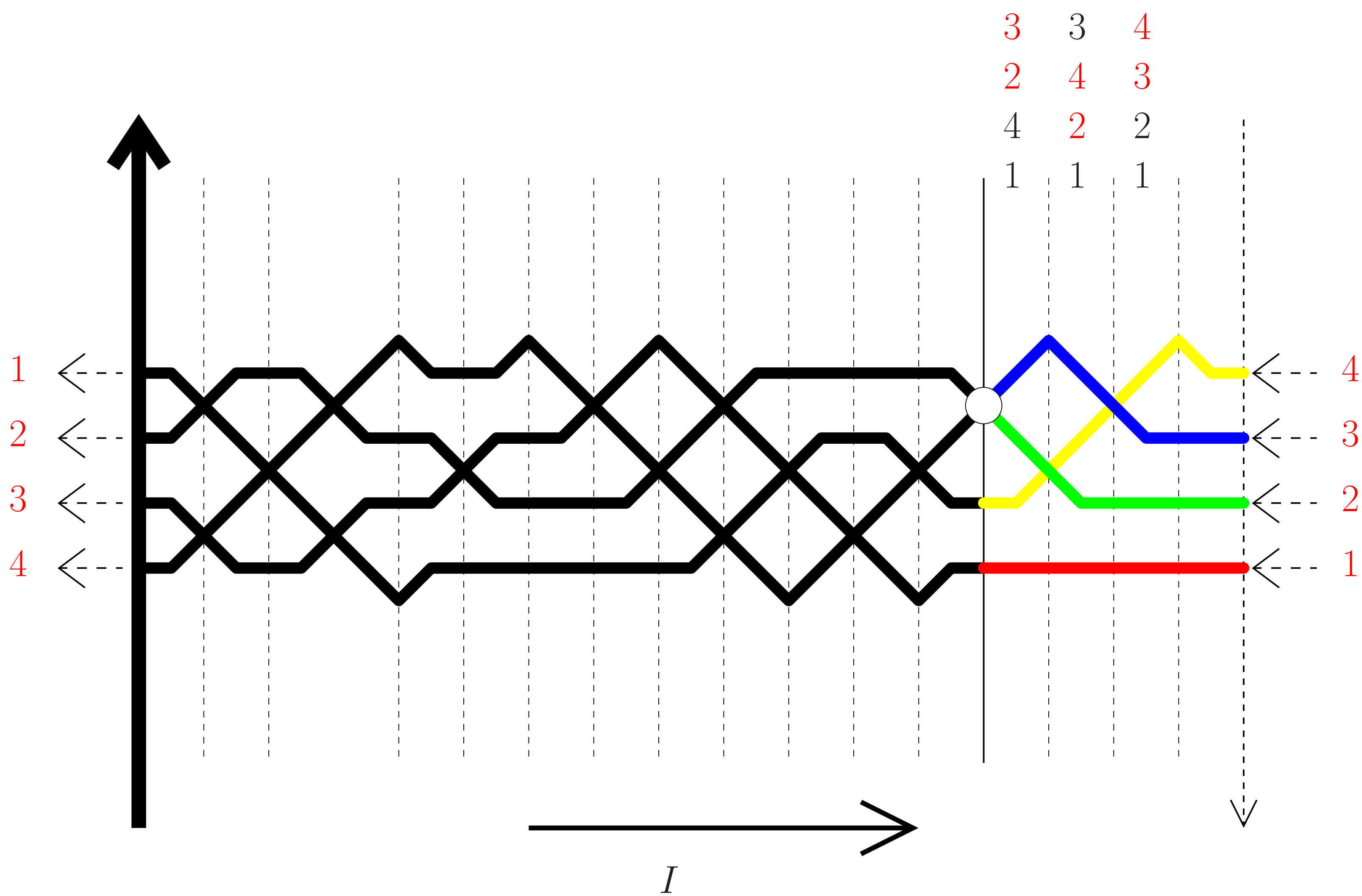

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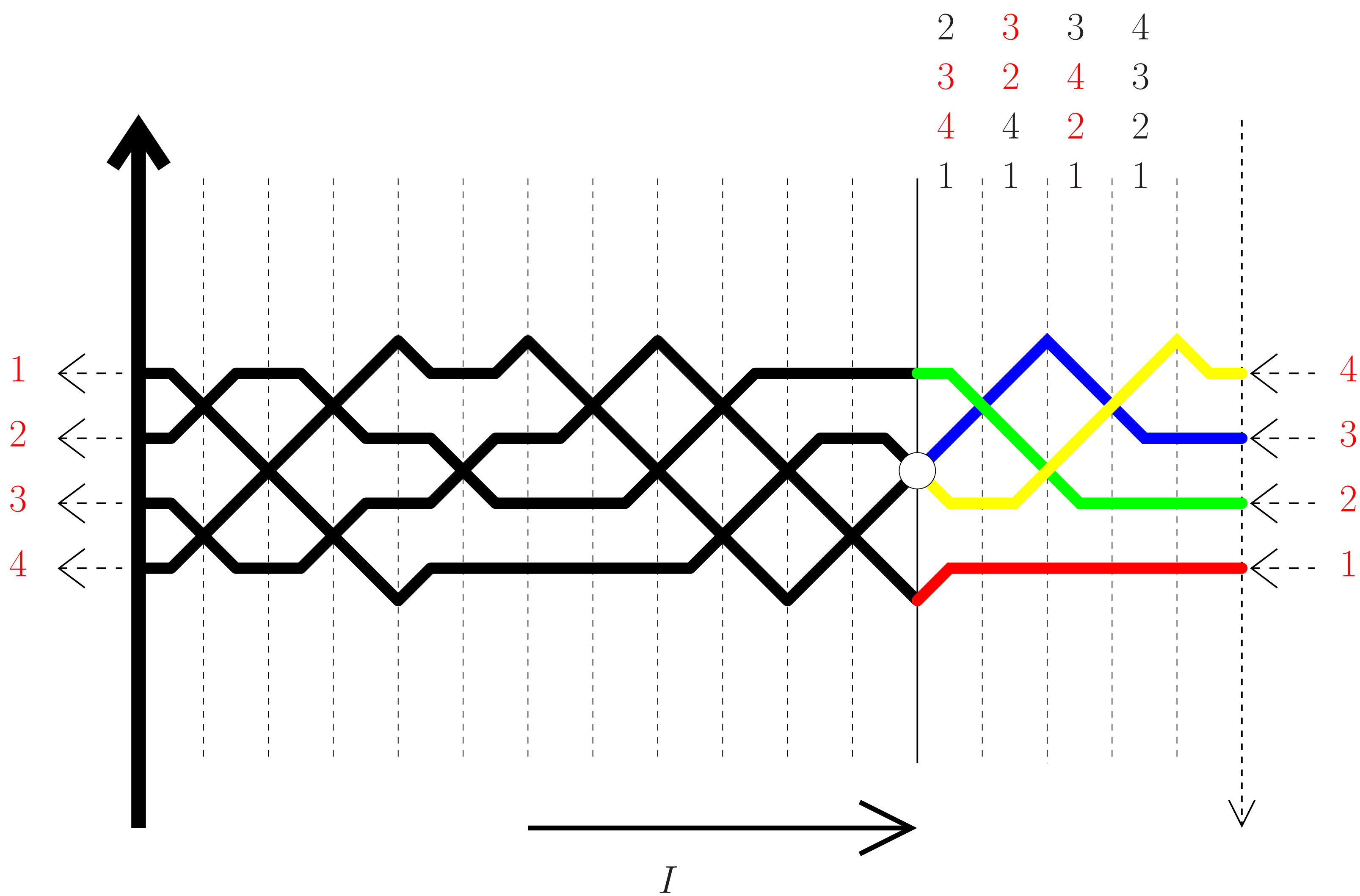

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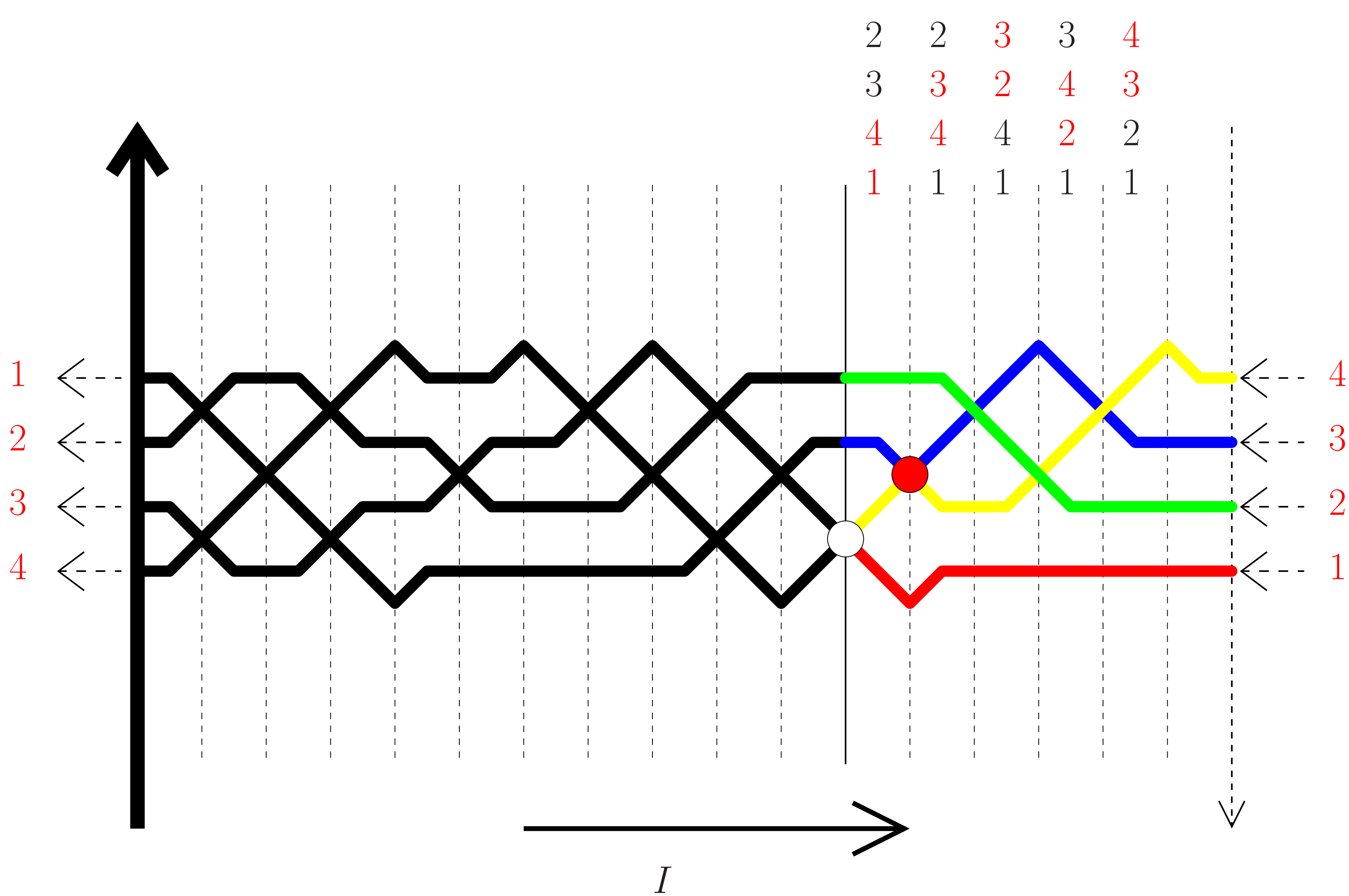

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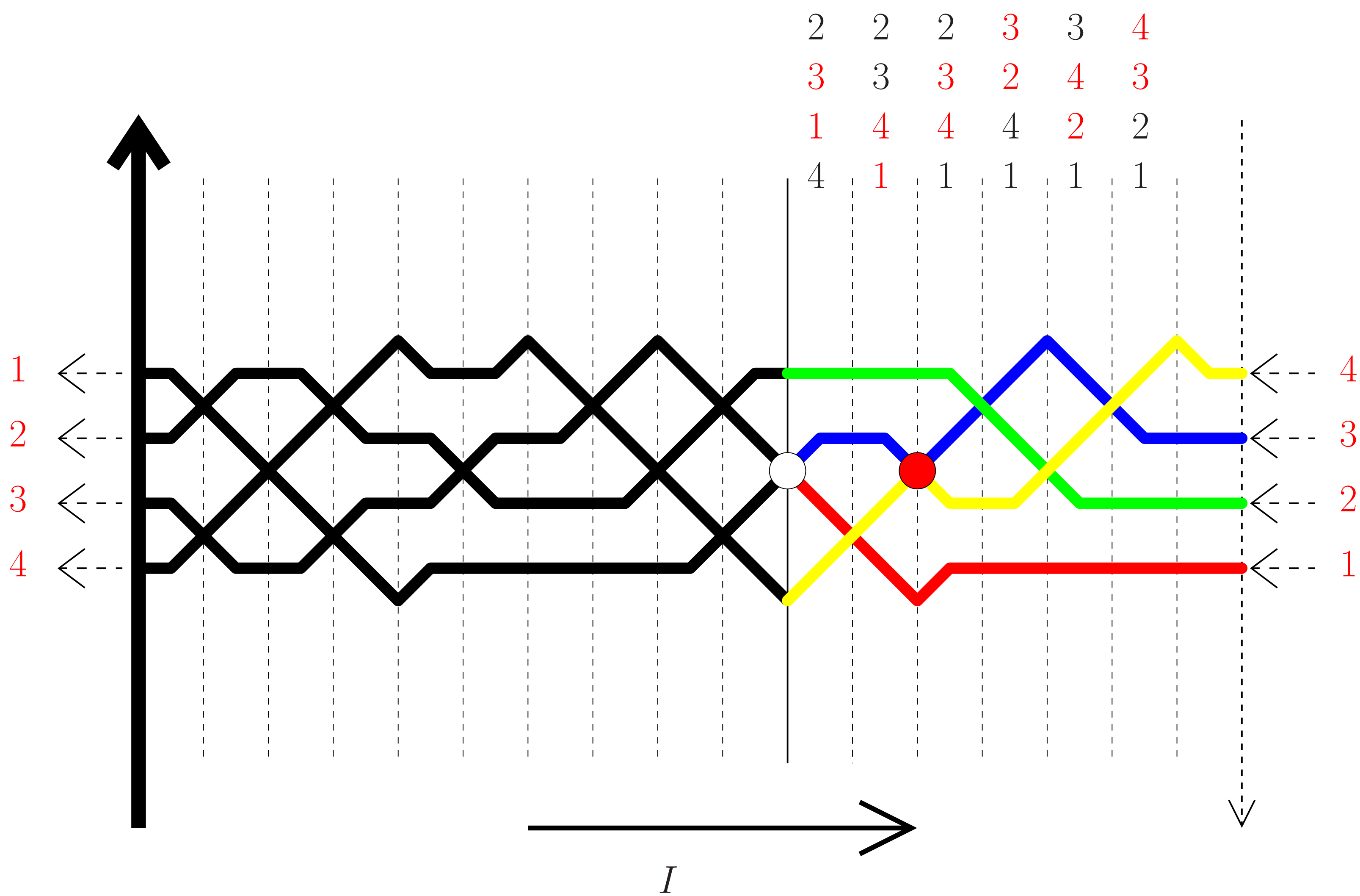

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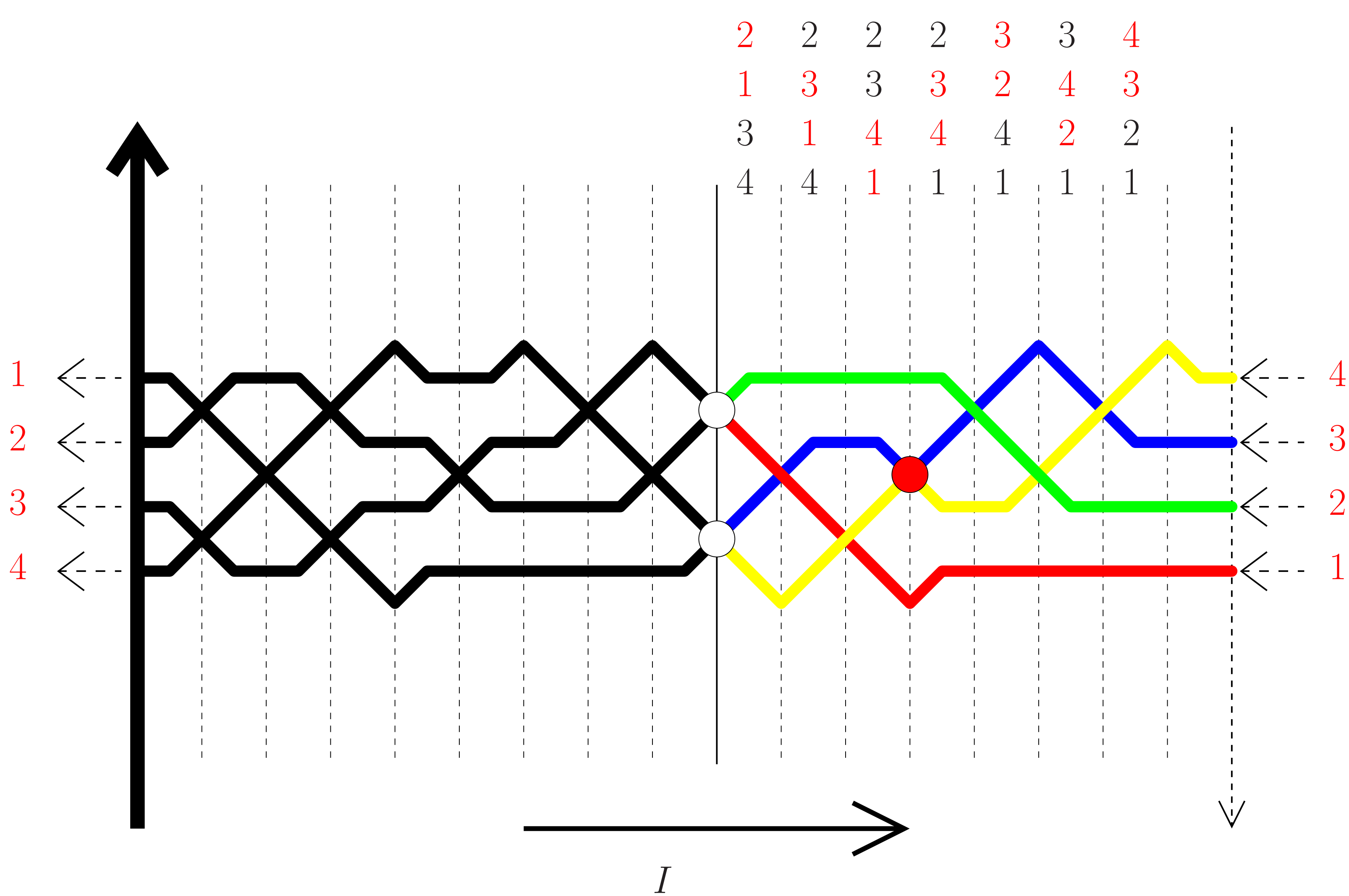

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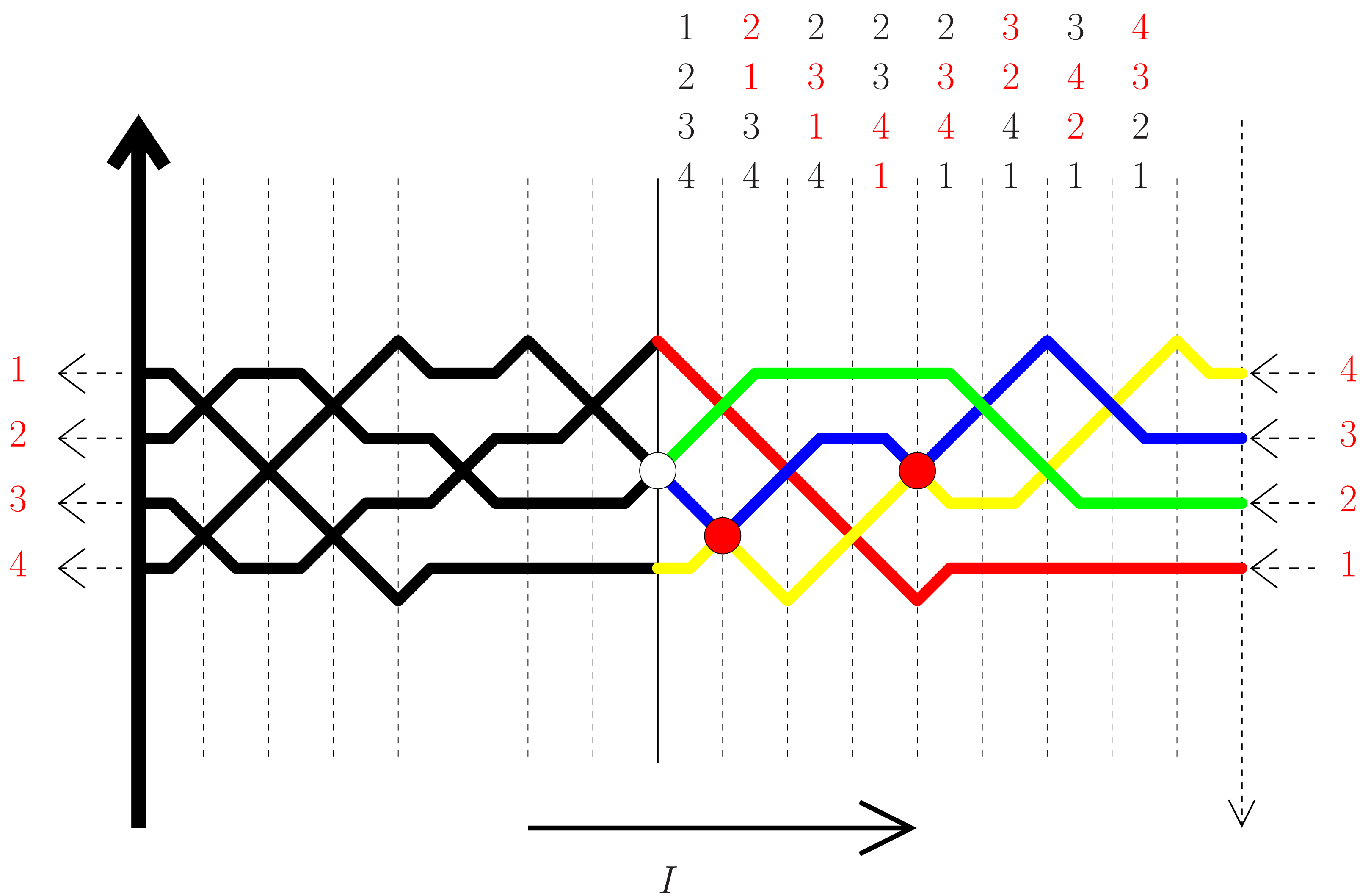

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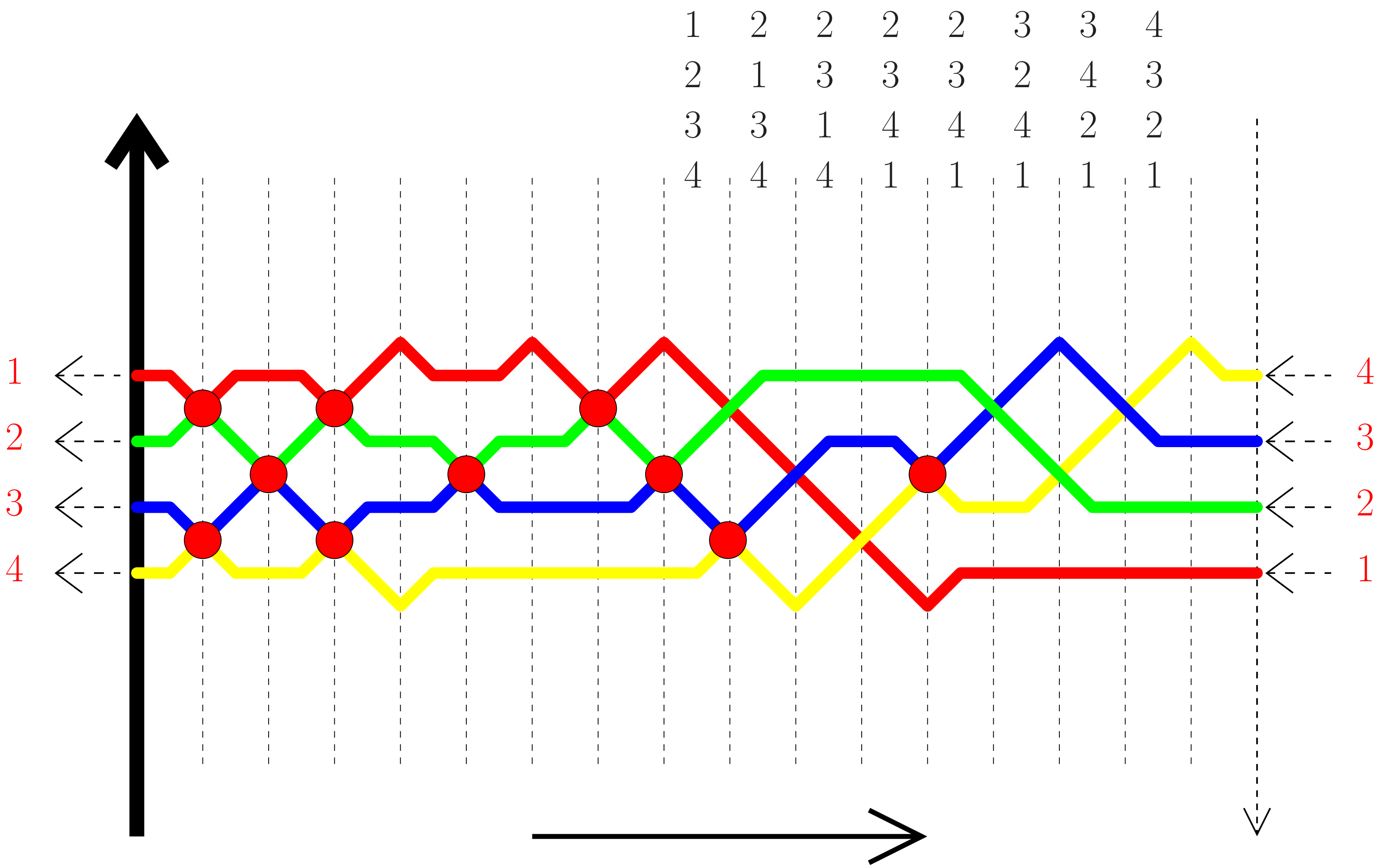

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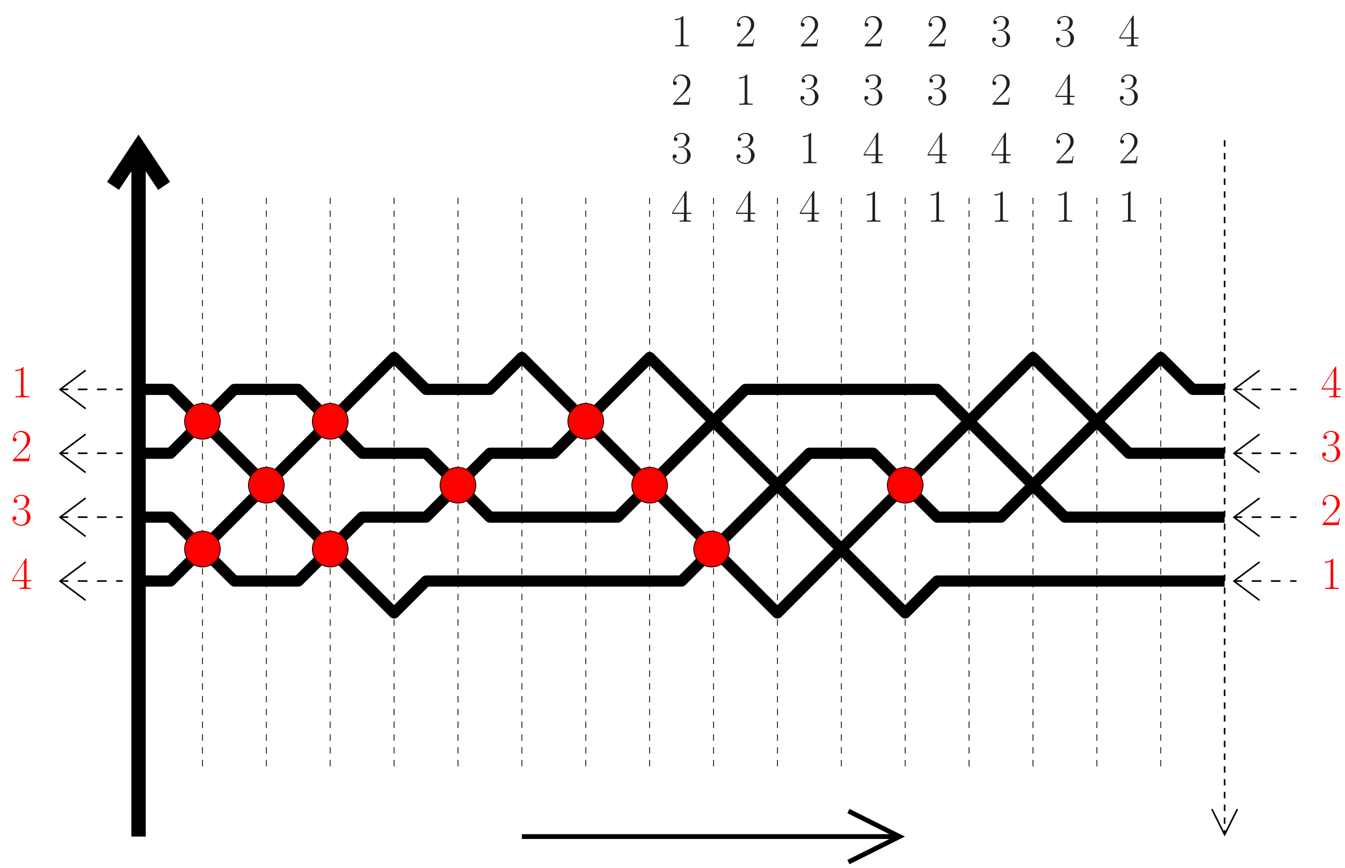

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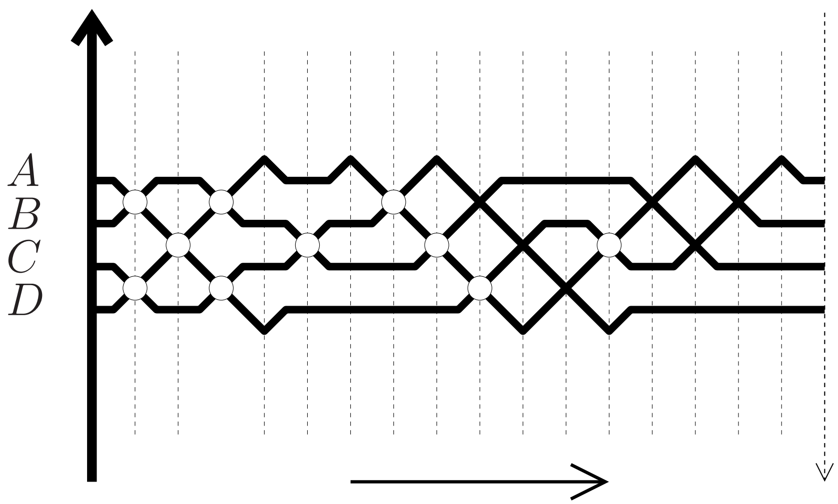
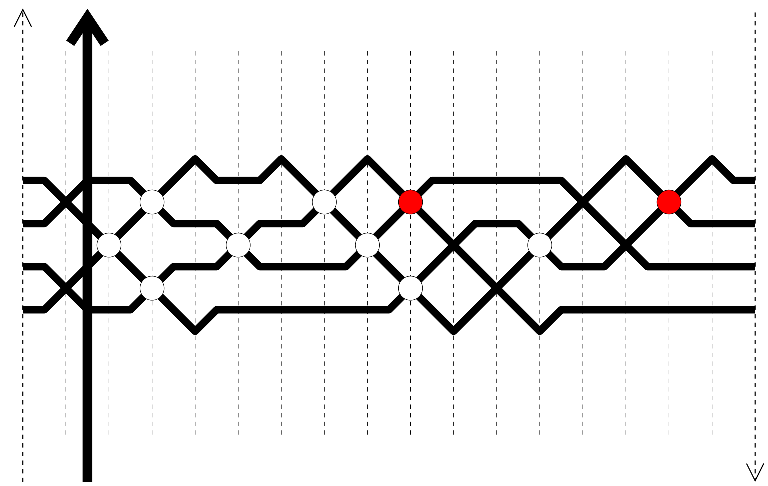
GREEDY FLIP PROPERTY



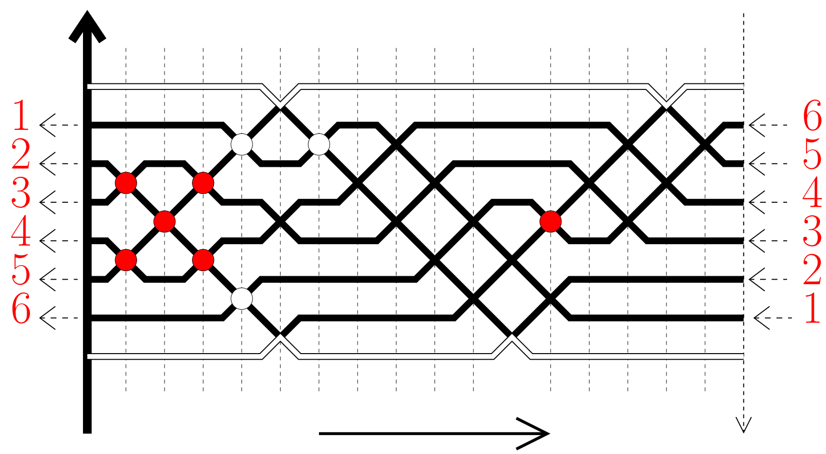


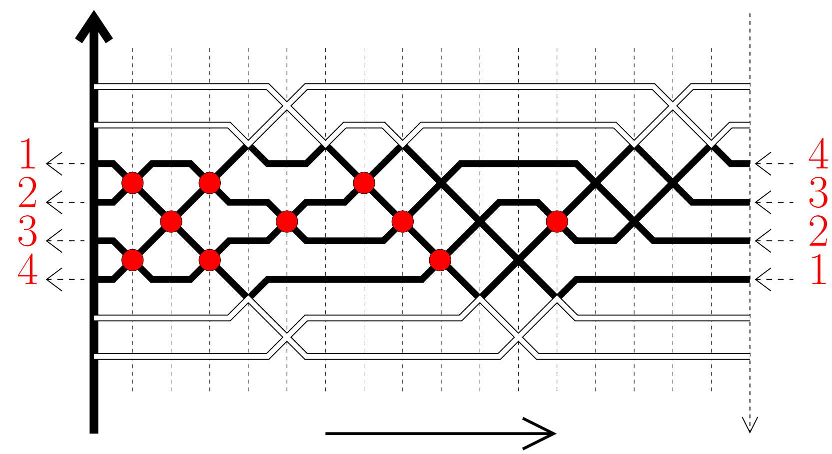
TH 10. *Let I be a filter and let M be a subset of the set of minimal elements of I. Then $G(I \setminus M)$ is obtained from $G(I)$ by flipping the elements of M.* □

P. & Vegter'95

Enumeration algorithm similar to the enumeration algorithm of pointed pseudotriangulations of Brönimann et al.'06.

ITERATION





TH 11. *Let $\mathcal{N}$ be a network and let $\mathcal{N}_1$ be the closure of $\mathcal{N}$ minus its first level. Then $\mathcal{N}_1$ is a network and for any filter I of $\mathcal{N}$ the contact points of $G(I;\mathcal{N})$ not on the first level of $\mathcal{N}$ are contact points of $G(I;\mathcal{N}_1)$.* □

TWO OPEN PROBLEMS

(1) Prove or disprove that the complex of atoms of a network is polytopal.

(2) Define a k-pseudotriangulation of an arrangement Γ of n double pseudolines as an atom of (the closure of) the support of Γ minus its k first levels. Design an $O(n \log n)$ algorithm to compute a 2-pseudotriangulation of an arrangement of n double pseudolines presented by its chirotope (Habert and P. 09)

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