The binary paint shop problem

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The binary paint shop problem

Definition

Instances of the binary paint shop problem PPW(2,1) are double occurrence words, i.e. words in which every letter occurs exactly twice, and every letter must be colored red once and blue once, so that the number of color changes is minimized.

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given word: ADEBAFCBCDEF

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ADEBAFCBCDEF   4 color changes

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ADEBAFCBCDEF 4 color changes

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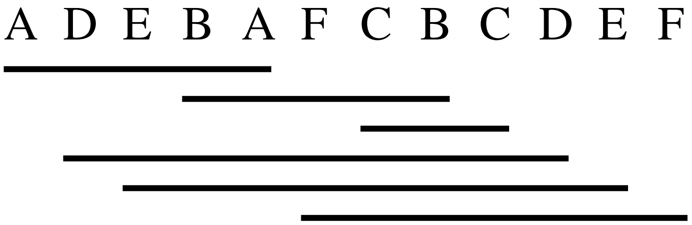
given word: ADEBAFCBCDEF

ADEBAFCBCDEF	4 color changes
ADEBAFCBCDEF	4 color changes
ADEBAFCBCDEF	2 color changes

Interval representation



Every interval must be cut an odd number of times.

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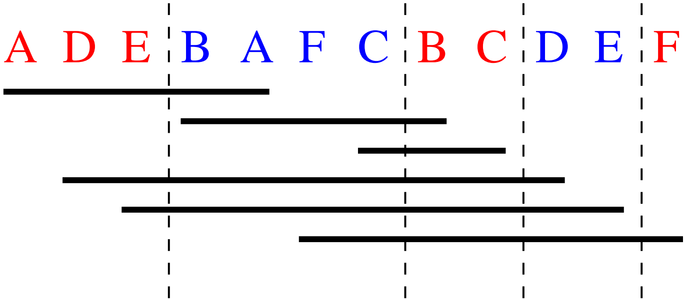

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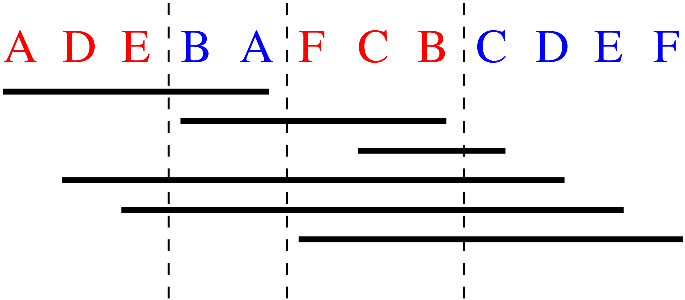

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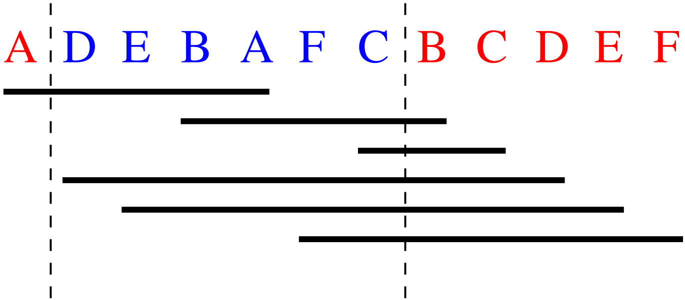

Every interval must be cut an odd number of times.

The complexity of the binary paint shop problem

Theorem (Bonsma, Epping, Hochstättler (06); Meunier, Sebő (09))

The binary paint shop problem is $\mathcal{APX}$-hard.

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Corollary

The binary paint shop decision problem is $\mathcal{NP}$-complete.

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Corollary

The binary paint shop decision problem is $\mathcal{NP}$-complete.

Problem

Is the binary paint shop problem in $\mathcal{APX}$, i.e. is there a (polynomial) constant factor approximation?

Greedy heuristic

Color the first letter red.

Scan the word from left to right, as long possible use the same color.

ABBCDECFGDFHIJKLHKAJIELG

Greedy heuristic

Color the first letter red.

Scan the word from left to right, as long possible use the same color.

AB BCDECFGDFHIJKLHKAJIELG

Greedy heuristic

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Scan the word from left to right, as long possible use the same color.

ABBCDE CFGDFHIJKLHKAJIELG

Greedy heuristic

Color the first letter red.

Scan the word from left to right, as long possible use the same color.

ABBCDECFGD FHIJKLHKAJIELG

Greedy heuristic

Color the first letter red.

Scan the word from left to right, as long possible use the same color.

ABBCDECFGDFHIJKL HKAJIELG

Greedy heuristic

Color the first letter red.

Scan the word from left to right, as long possible use the same color.

ABBCDECFGDFHIJKLHK AJIELG

Greedy heuristic

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Scan the word from left to right, as long possible use the same color.

ABBCDECFGDFHIJKLHKA JIELG

Greedy heuristic

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Optimality of the greedy heuristic

Observation

The greedy heuristic is optimal on instances that do not contain subwords of the form ABBA.

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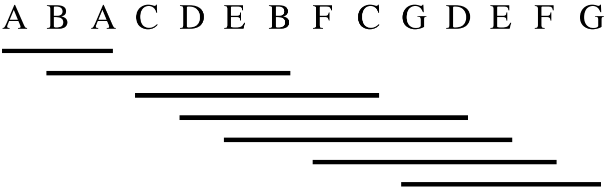

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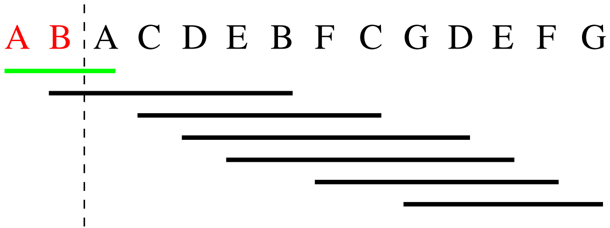

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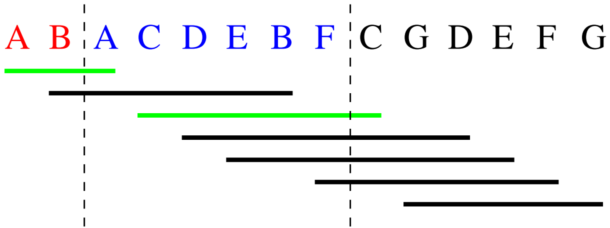

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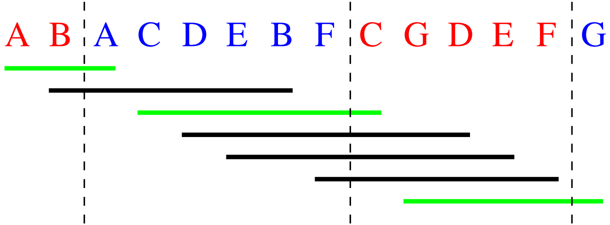

Optimality and perfectness of the greedy heuristic

Theorem (Amini, Meunier, Michel, Mohajeri (2010))

The greedy heuristic is optimal on instances that do not contain subwords of the form $ABACCB$ or $ABBCAC$.

Optimality and perfectness of the greedy heuristic

Theorem (Amini, Meunier, Michel, Mohajeri (2010))

The greedy heuristic is optimal on instances that do not contain subwords of the form ABACCB or ABBCAC.

Theorem (Rautenbach, Szigeti (2012))

The greedy heuristic is optimal on every subword of a word w if and only if w does not contain subwords of the form ABACCB or ADDBCCAB or ADDCBCAB.

Expected number of color changes for the greedy

Theorem (Amini,Meunier, Michel, Mohajeri (2010))

The expected number of color changes for the greedy heuristic on uniformly distributed double occurrence words of the length $2n$ with n characters is

$$\mathbb{E}_n(g) \leq \frac{2}{3}n.$$

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The expected number of color changes for the greedy heuristic on uniformly distributed double occurrence words of the length $2n$ with n characters is

$$\mathbb{E}_n(g) \leq \frac{2}{3}n.$$

Conjecture (Amini,Meunier, Michel, Mohajeri (2010))

$$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(g) = \frac{1}{2}.$$

On the proof of the conjecture I

Idea: The greedy heuristic ignors the first occurrence of a letter.

ABCZDBEDAECZ

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Idea: The greedy heuristic ignors the first occurrence of a letter. Consider the last letter Z of a word.

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Idea: The greedy heuristic ignors the first occurrence of a letter. Consider the last letter Z of a word. If we delete both occurrences of Z, then greedy colors the rest of the word in the same way.

ABC DBEDAEC

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Idea: The greedy heuristic ignors the first occurrence of a letter. Consider the last letter Z of a word. If we delete both occurrences of Z, then greedy colors the rest of the word in the same way.

Only the second occurrence of Z can create an additional color change.

ABCDBE DAEC

On the proof of the conjecture I

Idea: The greedy heuristic ignors the first occurrence of a letter. Consider the last letter Z of a word. If we delete both occurrences of Z, then greedy colors the rest of the word in the same way.

Only the second occurrence of Z can create an additional color change. This happens in about a half of the cases.

ABCDBEZDAECZ

On the proof of the conjecture II

Lemma

The greedy heuristic colors the first occurrence of Z red with probability $\frac{n}{2n-1}$ resp. blue with probability $\frac{n-1}{2n-1}$.

On the proof of the conjecture II

Lemma

The greedy heuristic colors the first occurrence of Z red with probability $\frac{n}{2n-1}$ resp. blue with probability $\frac{n-1}{2n-1}$.

Proof. First occurrence of Z

- at the beginning: Z red — 1 case
- after red letter: Z red — $n-1$ cases
- after blue letter: Z blue — $n-1$ cases

First Z red $\boxed{\frac{n}{2n-1}}$, first Z blue $\boxed{\frac{n-1}{2n-1}}$

Theorem (A, Hochstättler (2011)) The expected number of color changes for the greedy heuristic on uniformly distributed double occurrence words of length $2n$ with n letters is

$$\mathbb{E}_n(g) = \sum_{k=0}^{n-1} \frac{2k^2-1}{4k^2-1}.$$

First Z red $\boxed{\frac{n}{2n-1}}$, first Z blue $\boxed{\frac{n-1}{2n-1}}$

Theorem (A, Hochstättler (2011)) The expected number of color changes for the greedy heuristic on uniformly distributed double occurrence words of length $2n$ with n letters is

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$$\mathbb{E}_n(g) = \mathbb{E}_{n-1}(g) + \frac{n}{2n-1} \cdot \frac{n-2}{2n-3} +$$

First Z red $\frac{n}{2n-1}$, first Z blue $\frac{n-1}{2n-1}$

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First Z red $\frac{n}{2n-1}$, first Z blue $\frac{n-1}{2n-1}$

Theorem (A, Hochstättler (2011)) The expected number of color changes for the greedy heuristic on uniformly distributed double occurrence words of length $2n$ with n letters is

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&= \sum_{k=0}^{n-2}\frac{2k^2-1}{4k^2-1} + \frac{2(n-1)^2-1}{4(n-1)^2-1} = \sum_{k=0}^{n-1}\frac{2k^2-1}{4k^2-1}.
\end{aligned}$$

On the performance of the greedy heuristic

$$A_1 \dots A_k A_k A_{k+1} \dots A_{2k} A_{2k} A_1 A_{k+1} A_2 A_{k+2} \dots A_{k-1} A_{2k-1}$$

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$$\color{red}{A_1 \dots A_k}\ A_k A_{k+1} \dots A_{2k} A_{2k} A_1 A_{k+1} A_2 A_{k+2} \dots A_{k-1} A_{2k-1}$$

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Greedy: n color changes; Optimal: 3 color changes

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Greedy: n color changes; Optimal: 3 color changes

$\Longrightarrow$ Greedy is not a constant factor approximation

The Red-first heuristic and its performance

Color the first occurrence of every letter red

$$A_1 \dots A_k A_k A_{k+1} \dots A_{2k} A_{2k} A_1 A_{k+1} A_2 A_{k+2} \dots A_{k-1} A_{2k-1}$$

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on the worst-case instance of greedy red-first colors optimally!

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red-first needs $n+1$ color changes; greedy/optimal coloring needs 2

$\longrightarrow$ Red-first is neither a constant factor approximation

Result for the red-first heuristic

Theorem (A, Hochstättler (2011))

The expected number of color changes for the red-first heuristic on an instance of length $2n$ is

$$\mathbb{E}_n(rf) = \frac{2n+1}{3}.$$

Idea to improve the greedy heuristic

$$AZABBZ$$

Idea to improve the greedy heuristic

$\color{red}{AZ}$ $ABBZ$

Idea to improve the greedy heuristic

$AZAB\ BZ$

Idea to improve the greedy heuristic

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Recursive greedy heuristic:

$AZABBZ$

Idea to improve the greedy heuristic

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Recursive greedy heuristic:

$\longrightarrow$ delete both occurrences of the last letter Z

$A \quad ABB$

Idea to improve the greedy heuristic

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Recursive greedy heuristic:

⟶ delete both occurrences of the last letter Z

⟶ color the rest recursively

$$A \quad A$$

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Recursive greedy heuristic:

$\longrightarrow$ delete both occurrences of the last letter Z

$\longrightarrow$ color the rest recursively

$\longrightarrow$ if the first Z is in a color change: color in such a way that no additional color changes is created; otherwise color according to greedy

$$A \quad A$$

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Recursive greedy heuristic:

$\longrightarrow$ delete both occurrences of the last letter Z

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Recursive greedy heuristic:

⟶ delete both occurrences of the last letter Z

⟶ color the rest recursively

⟶ if the first Z is in a color change: color in such a way that no additional color changes is created; otherwise color according to greedy

$AZABBZ$

Result for the recursive greedy heuristic

Theorem (A, Hochstättler (2011))

For all $n \geq 1$, the expected number $\mathbb{E}_n(rg)$ of color changes for the recursive greedy heuristic is bounded by

$$\frac{2}{5}n + \frac{8}{15} \leq \mathbb{E}_n(rg) \leq \frac{2}{5}n + \frac{7}{10}.$$

Summary

red-first heuristic	$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(rf)$	$=$	$\frac{2}{3}$
greedy heuristic	$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(g)$	$=$	$\frac{1}{2}$
recursive greedy heuristic	$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(rg)$	$=$	$\frac{2}{5}$

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recursive greedy heuristic	$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(rg)$	$=$	$\frac{2}{5}$

Problem

Find better heuristics (with expected number of color changes $\leq 2n/5$).

Summary

red-first heuristic	$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(rf) = \frac{2}{3}$
greedy heuristic	$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(g) = \frac{1}{2}$
recursive greedy heuristic	$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_n(rg) = \frac{2}{5}$

Problem

Find better heuristics (with expected number of color changes $\leq 2n/5$).

Problem

Characterize the instances where the recursive greedy is optimal.

Optimal coloring

Problem

Determine the expected number of color changes for optimal coloring.

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Determine the expected number of color changes for optimal coloring.

n	1	2	3	4	5	6	7
$\mathbb{E}(opt)$	1	$\frac{4}{3}$	$\frac{26}{15}$	$\frac{223}{105}$	$\frac{2355}{945}$	$\frac{29541}{10395}$	$\frac{429677}{135135}$
$\frac{\mathbb{E}(opt)}{n}$	1	0.6667	0.5778	0.5310	0.4984	0.4736	0.4542

Optimal coloring

Problem

Determine the expected number of color changes for optimal coloring.

n	1	2	3	4	5	6	7
$\mathbb{E}(opt)$	1	$\frac{4}{3}$	$\frac{26}{15}$	$\frac{223}{105}$	$\frac{2355}{945}$	$\frac{29541}{10395}$	$\frac{429677}{135135}$
$\frac{\mathbb{E}(opt)}{n}$	1	0.6667	0.5778	0.5310	0.4984	0.4736	0.4542

Conjecture (Meunier, Neveu (2012))

This number is sublinear in n.

Optimal coloring and heuristics

Remark

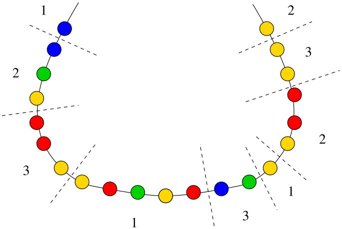
In case

- *there is a constant factor approximation and*
- *the expected number of color changes for optimal coloring is sublinear*

every constant factor approximation for the binary paint shop problem is (in expectation) a better heuristic than the recursive greedy.

The necklace splitting problem

Given: open necklace of length n with t types of beads, every type i occurs qa_i times, q thieves want to cut the necklace in a fair way, so that everyone receives exactly a_i beads of every type i.



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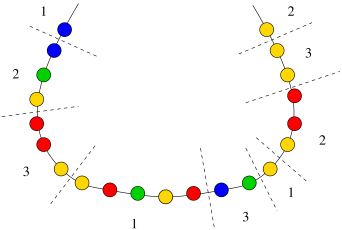

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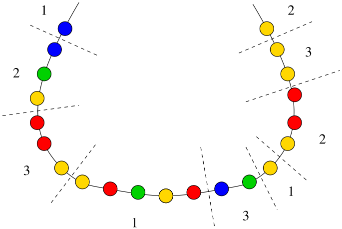

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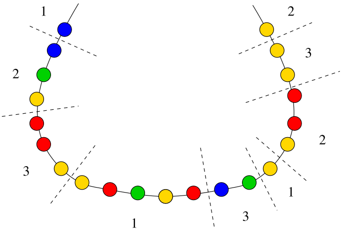

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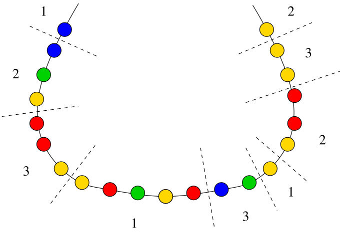

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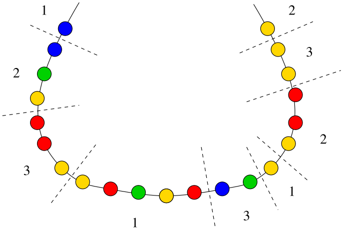

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Problem (Meunier, Neveu (2012))

Is the necklace splitting problem PPAD-complete for $q = 2$?

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Thank you!