Exercises for Combinatorial and Computational Geometry

Homework #5 - Voronoi diagrams and arrangements

deadline 5.1.2015

1. Let P be a finite point set in the plane with no three points collinear and no four points cocircular. Define a graph DT (called *Delaunay triangulation*) on P as follows: two points a, b are connected by an edge if and only if there exists a circular disk with both a and b on the boundary and no point of P in its interior.

   (a) Prove that DT is a triangulation — a plane graph where every face except of the outer-face is a triangle. [**3**]

   (b) Prove that DT is the dual graph to the graph of the Voronoi diagram of the set P. [**3**]

2. Arrangement A of hyperplanes in $\mathbb{R}^d$ is *simple*, if the intersection of every k hyperplanes from A is $(d-k)$-dimensional for $k = 2, 3, \ldots, d+1$. For $k = 0, 1, \ldots, d$, prove that the number of k-faces in a simple arrangement of hyperplanes in $\mathbb{R}^d$ equals [**3**]

$$\sum_{i=d-k}^{d} \binom{n}{i} \binom{i}{d-k}.$$

3. Let $P = \{p_1, p_2 \ldots p_n\}$ be a point set in the plane. Let us say that points x,y have the *same view* of P if the points of P are visible in the same cyclic order from them. If rotating light rays emanate from x and from y, the points of P are lit in the same order by these rays. We assume that neither x nor y is in P and that neither of them can see two points of P in occlusion.

   Show that the maximum possible number of points with mutually distinct views of P is $O(n^4)$. [**2**]

4. (a) How many d-dimensional cells are there in the arrangement of the $\binom{d}{2}$ hyperplanes in $\mathbb{R}^d$ with equations $x_i = x_j$, where $1 \leq i < j \leq d$? [**3**]

   (b) How many d-dimensional cells are there in the arrangement of the $\binom{d}{2}$ hyperplanes in $\mathbb{R}^d$ with equations $x_i + x_j = 0$ and $x_i = x_j$ for all $1 \leq i < j \leq d$? [**2**]

Information about the practicals can be found at `http://kam.mff.cuni.cz/kvg`