Exercises for Combinatorial and Computational Geometry

Homework #1 – Convex sets

hints 27.10.2014, deadline 3.11.2014

Please choose a nickname that will be used in the list of scores on the webpage of the practicals. If you have already chosen a nickname, you can just sign your solutions with either your name or your nickname.

1. Prove that the convex hull of every bounded closed set $M \subset \mathbb{R}^2$ is closed. **[2]**
2. Prove Carathéodory's theorem (you may use Radon's lemma). **[2]**
3. Let $M = \{x_1, y_1, x_2, y_2, ..., x_{d+1}, y_{d+1}\}$ be a set of points in $\mathbb{R}^d$. Prove that M can be partitioned into two subsets A and B such that each one of these subsets contains exactly one point from $\{x_i, y_i\}$ for every $i = 1, 2, ..., d+1$ and the convex hulls of A and B have a nonempty intersection. (You may use the fact that the $(d+1)$-tuple of vectors $x_i - y_i$ is linearly dependent and then use an approach similar to the proof of Radon's theorem.) **[2]**
4. Let M be a finite set of at least four points from the plane that are colored red and blue. For every 4-tuple V of points from M there is a line which strictly separates red points of V from the blue points of V. Prove that it is possible to strictly separate all the red points from M from the blue points of M. **[3]**
5. Let $X_1, X_2, \ldots, X_{d+1}$ be finite point sets in $\mathbb{R}^d$ such that the origin lies in $\mathrm{conv}(X_i)$ for every $i \in \{1, 2, \ldots, d+1\}$. Prove that there exist $(d+1)$ points $x_i \in X_i$, $i \in \{1, 2, \ldots, d+1\}$, such that the origin lies in $\mathrm{conv}(\{x_1, x_2, \ldots, x_{d+1}\})$. **[4+hint]**

Informations about the practicals can be found here:`http://kam.mff.cuni.cz/kvg`