WP08 – Crossing number

This work package concerns the following natural problem in graph drawing: to find a drawing of a given graph that minimizes the number of pairwise edge crossings. This minimum, which obviously is a nonzero positive integer for any nonplanar graph, is called the *crossing number* of the graph. Investigation of graph crossing numbers has been initiated by Turán during WW II, and this topic has since found important applications, e.g., in VLSI design and graph visualization.

Despite decades of research, many basic questions remain unanswered – such as what are the crossing numbers of complete, complete bipartite, or toroidal grid graphs. This fact indicates the difficulty of the crossing number problem, and the problem indeed is very hard from both the theoretical and algorithmic points of view. Even more, in comparison with other typical NP-hard graph problems (such as colouring or Hamiltonicity), crossing minimization appears practically much harder already on quite small graphs. In this respect it is worth to mention a recent involved (ILP-based) practically usable approach to exact crossing minimization by Chimani, Mutzel, and Bomze [2].

One may clearly see that the theory of graph crossing number has matured especially during the last decade, evolving into a rich mathematical theory with many new results and many more open problems of a fundamental nature. Some of the recent results have been trully groundbreaking: the separation between odd and ordinary crossing numbers by Pelsmajer, Schaefer, and Stefankovič [6], hardness of computation of the crossing number even for nearly-planar graphs by Cabello and Mohar [1], or a construction of an unexpected crossing-critical family by Dvořák and Mohar [4], to mention just a few. Yet many other fundamental questions in the area remain wide open.

The proposed workpackage leader, Petr Hliněný, has also remarkably contributed to the recent development in the area of crossing numbers. Out of his numerous related results we mention structural study of crossing-critical graphs [5] and subsequent constructions of interesting critical families, and a series of papers devoted to constant-factor approximation algorithms for the crossing number on restricted graph classes (in collaboration with Salazar, and with Chimani and Mutzel) such as most recent [3]. The existing intensive collaboration of the work package leader with Chimani and Mutzel is especially noticeable with respect to our CRP.

Milestones of work package WP08: M08.1 Improve the lower estimates on the crossing number of surface embedded graphs.
M08.2 Attempt to extend M08.1 to nonorientable surfaces.
M08.3 Lower estimates and approximations of the minor crossing number of surface embedded graphs.
M08.4 Try to determine the computational complexity of crossing number of almost planar graphs of bounded degree.
M08.5 Further investigation of crossing-critical graphs.
M08.6 Attempt to give an approximative solution for multiple-edge insertion to planar graphs.

References for workpackage WP08

[1] S. Cabello and B. Mohar. Adding one edge to planar graphs makes crossing number hard. In: *Proc. ACM Symp. Comput. Geom. (SoCG'10)*. To appear.
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[3] M. Chimani and P. Hliněný. Approximating the crossing number of graphs embeddable in any orientable surface. In: *Proc. ACM-SIAM Symp. Discrete Algorithms (SODA'10)*, p. 918–927, 2010.
[4] Z. Dvořák and B. Mohar. Crossing-critical graphs with large maximum degree. *J. Combin. Theory Ser. B*, 100:413–417, 2010.
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