Corruption Detection on Networks

Noga Alon, Elchanan Mossel and Robin Pemantle

presented by Radek Hušek

Definition 1 (δ-good expander)**.** *Let $\delta < 1/8$. Graph $G = (V, E)$ on n vertices is δ-good expander if following holds:*

- $\forall U \subset V$ *such that* $|U| \leq 2\delta n$ *holds* $|N(U) \setminus U| > |U|$
- $\forall A, B \subset V$ *such that* $|A| \geq \delta n$ *and* $|B| \geq n/4$ *there exists an edge between* A *and* B*.*

Standard results imply that random d-regular graph are δ-good expanders with high probability (where d depends on δ).

Definition 2 (Directed δ-good expander)**.** *Let $\delta < 1/16$. Digraph $G = (V, E)$ on n vertices is δ-good directed expander if following holds:*

- $\forall U \subset V$ *such that* $|U| \leq 4\delta n$ *holds* $|N^+(U) \setminus U| > |U|$
- $\forall A, B \subset V$ *such that* $|A| \geq \delta n$ *and* $|B| \geq n/4$ *there exist both an edge from* A *to* B *and an edge from* B *to* A*.*

Main results

Theorem 3 (Tractability for expanders)**.** *Let $G = (T \dot\cup B, E)$ be a δ-good expander and suppose $|T| > |B|$. Then when getting reports of each vertex of G about all its neighbors we can identify a subset $T' \subseteq T$ and a subset $B' \subseteq B$ so that $|T' \cup B'| \geq (1 - \delta)n$.*

Moreover, if $|T| > (1/2 + \delta)n$ then T' and B' can be computed from given reports in a linear time.

Theorem 4 (NP-hardness)**.** *For any $\delta > 0$ there exists a $\gamma > 0$ such that following promise problem is NP-hard. The input is a δ-good expander $G = (V, E)$ on n vertices and all the reports of vertices about their neighbors. The promise is that either*

- *there exists partition of $V = T \dot\cup B$ which is consistent with all the reports and $|T| \geq n/2 + \gamma n$, or*
- *all partitions $V = T \dot\cup B$ which are consistent with reports satisfy $|T| \leq n/2 - \gamma n$.*

The objective is to distinguish between the two options above.

Theorem 5 (Non-tractability for graphs with small separators)**.** *Let $G = (V, E)$ be a graph on n vertices such that it is possible to remove at most εn vertices and get a graph in which any connected component is of size at most εn. Then even knowing that $|T| \geq (1 - 2\varepsilon)n$ there is no deterministic algorithm that identifies even single member of T given all the reports. In particular, this is the case for planar graphs and more generally graphs with fixed excluded minors even if $\varepsilon = \Theta(1/\sqrt[3]{n})$.*

Directed version

Lemma 6 (Existence of δ-good directed expanders)**.** *There are absolute positive constants c_1, c_2 so that for any fixed positive $\delta < 1/16$ there is a constant $d < c_1/\delta$ and infinitely many values of n for which there is a δ-good directed expander on n vertices in which total degree of each vertex is d and there is no cycle shorter than $c_2 \log n / \log d$ (of any orientation).*

Theorem 7 (Tractability for directed expanders)**.** *Let $G = (T \dot{\cup} B, E)$ be a δ-good directed expander and suppose $|T| > |B|$. Then when getting reports of each vertex of G about all its out-neighbors we can identify a subset $T' \subseteq T$ and a subset $B' \subseteq B$ so that $|T' \cup B'| \geq (1 - \delta)n$.*

Moreover, if $|T| > (1/2 + 2\delta)n$ then T' and B' can be computed from given reports in a linear time.