- *weighted* – let T be a (theorem, definition, ...), T' is *weighted* T if T' arises from T by replacing $\{0,1\}$ with non-negative reals (possibly ≤ 1).
- *measure* – if μ is called *measure*, think of it as of weighted characteristic function.
- *pseudorandom* – a set $A \subseteq X$ is f-*pseudorandom*, if $\mathbf{E}[f(x)\colon x \in A]$ behaves as if A was a random set.
- *relative* – let T be a weighted theorem, T' is *relativized* T if T' arises from T by replacing $0 \leq f(x) \leq 1 \forall x$ by $0 \leq f(x) \leq \mu(x) \forall x$, μ is pseudorandom.
- *complexity* – let G be a bipartite graph, its complexity is the minimum number of complete bipartite graphs such that G is their union.
- *removal* – let O be an object and f a function such that $\mathbf{E}[f\colon O] < \delta$, then there exists O' such that $\mathbf{E}[f\colon O'] = 0$, $\mathbf{E}[O \setminus O'] < \varepsilon$ and $O \setminus O'$ has small complexity.

Theorem 1 (Szemerédi)**.** *$\forall k \forall \delta > 0 \exists c \exists N_0 \forall N > N_0 \forall A \subseteq Z_N$ such that $\mathbf{E}[1_A] \geq \delta$ is*

$$\mathbf{E}[1_A(x) \cdot 1_A(x+d) \cdots 1_A(x+(k-1)d)\colon x, d \in Z_N] > c.$$

Definition. (Linear form) $\mu\colon Z_N \to \mathbb{R}_{\geq 0}$ satisfies k-linear forms, if

$$\mathbf{E}\Big[\prod_{j=1}^{k} \prod_{\omega \in \{0,1\}^{[k]\setminus j}} \mu\Big(\sum_{i=1}^{k}(i-j)x_{i,\omega_i}\Big)\colon x_{1\ldots k,\{0,1\}} \in Z_N\Big] = 1 + o(1),$$

and the same holds for all subexpressions (i.e., some $\mu(\ldots)$ omitted).

Theorem 2 (Relative Szemerédi)**.** *(as above), $\forall \mu, f\colon Z_N \to \mathbb{R}_{\geq 0}$, $\mathbf{E}[f] \geq \delta$, $f \leq \mu$, μ satisfies k-linear forms, (as above).*

Definition. (Hypergraph system) $(J, V_{j\in J}, r, H)$ is a h.s. for J finite set (graph is J-partite), V_j's are finite vertex sets, and H is an r-uniform hypergraph ("template graph"). For edge e define $V_e = \prod_{j\in e} V_j$, weight function g is a collection of functions $g_e\colon V_e \to \mathbb{R}_{\geq 0}$.
Linear forms for hypergraphs are as before, but we are counting $\mu_e(x_{e,\omega})$ instead.
Skeleton of an edge e is $\partial e = \{f\colon f \subseteq e, |f| = |e| - 1\}$.

Theorem 3 (Relative hypergraph removal lemma)**.** *Let $(J, V_{j\in J}, r, H)$ be a hypergraph system with weight function g in $[0,1]$. If $\mathbf{E}[\prod_{e\in H} g_e(x_e)\colon x \in V_J] < \delta$, then there exists a weight function g' in $\{0,1\}$, such that $\mathbf{E}[\prod_{e\in H} g'_e(x_e)] = 0$, g'_e has low complexity and $\mathbf{E}[g_e(x_e) \cdot (1 - g'_e(x_e)] \leq \varepsilon$.*

Definition. (Regular pair) Two functions g_e, $\tilde{g}_e\colon V_e \to \mathbb{R}_{\geq 0}$ form an ε-regular pair, if

$$\forall B_f \subseteq V_f \quad \Big|\mathbf{E}\big[(g_e(x_e) - \tilde{g}_e(x)) \prod_{f\in\partial e} 1_{B_f}(x_f)\colon x_e \in V_e\big]\Big| < \varepsilon.$$

Theorem 4 (Sparse hypergraph regularity lemma)**.** *For every $\varepsilon > 0$, every $g\colon V_{[r]} \to \mathbb{R}_{\geq 0}$ a function which is $\gamma(\varepsilon)$-upper regular, there exists a function $\tilde{g}$ with complexity at most $\zeta(r, \varepsilon)$ such that $(g, \tilde{g})$ is ε-regular.*

Theorem 5 (Relative counting regularity lemma)**.** *For every $\gamma > 0$, finite J there exists $\varepsilon > 0$, such that the following is true. Let $(J, V_{j\in J}, r, H))$ be a hypergraph system, $g, \tilde{g}, \mu$ weight functions such that μ satisfies H-linear forms conditions, $0 \leq g \leq \mu$, $0 \leq \tilde{g} \leq 1$, $(g, \tilde{g})$ is ε-regular pair; then*

$$\Big|\mathbf{E}\big[\prod_{e\in H} g_e(x_e)\colon x \in V_J\big] - \mathbf{E}\big[\prod_{e\in H} \tilde{g}_e(x_e)\colon x \in V_J\big]\Big| < \gamma.$$