The condensation transition in random hypergraph 2-coloring

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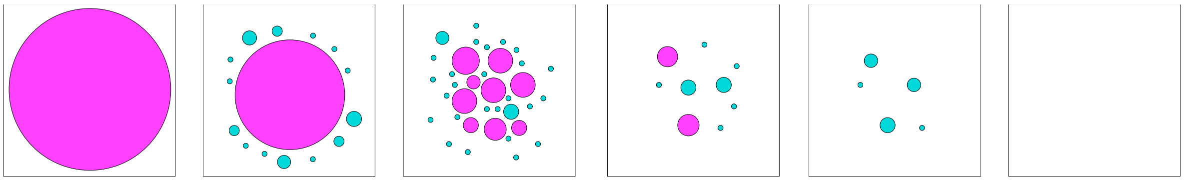

Theorem *(Friedgut, Kalai): For every symmetric monotone A, if:*

$$P[G(n,p) \text{ satisfies } A] \geq \varepsilon$$

then:

$$P[G(n,q) \text{ satisfies } A] > 1-\varepsilon$$

for $q = p + C \cdot \dfrac{\log(1/2\varepsilon)}{\log(n)}$, *where C is universal constant.*

We fix $k \geq k_0$. The theorem implies the existence of critical density r_{col}, such that a k-uniform hypergraph with fewer than $r_{col} \cdot n$ is 2-colorable w.h.p. and graphs with every higher density are not 2-colorable w.h.p.

Previous best bounds on r_{col} were established using first and second moment method:

$$2^{k-1}\ln 2 - \frac{1+\ln 2}{2} + o_k(1) = r_{second} \leq r_{col} \leq r_{first} = 2^{k-1}\ln 2 - \frac{ln 2}{2} + o_k(1).$$

The authors refine the second moment method to improve the lower bound by additive constant (roughly 0.15).

Theorem *1.1: For all $k \geq k_0$ and $r < r_{cond} = 2^{k-1}\ln 2 - \ln 2$ the random hypergraph $H_k(n,m)$ is 2-colorable w.h.p. and*

$$\ln(Z) \sim \ln \mathbb{E}[Z].$$

Furthermore, they show that there is an interval of densities after r_{cond}, dubbed "condensation phase" by physicists, where a random graph is still 2-colorable and the nature of the solutions changes dramatically.

Theorem *1.3: For all $k \geq k_0$ there exists $\varepsilon_k(\varepsilon_k \to 0), \delta_k > 0, \zeta_k \geq 0$ such that the following two statements are true.*

1. *W.h.p. $H_k(n,m)$ is 2-colorable for all $r < r_{cond} + \epsilon_k + \delta_k$.*

2. *For any density r with $r_{cond} + \varepsilon_k < r < r_{col}$ we have:*

$$\ln(Z) < \ln \mathbb{E}[Z] - \zeta_k \cdot n \qquad w.h.p.$$