

$L(v_1, \dots, v_n)$	lattice: $\{\sum_i a_i v_i \mid a_i \in \mathbf{Z}\}$, where v_1, \dots, v_n independent in \mathbf{Z}^k
CVP	IN: $(v_1, \dots, v_n), y \in \mathbf{R}^k$ OUT: $v \in L(v_1, \dots, v_n)$ GOAL: minimize $\ l - y\ _1$
SVP	IN: (v_1, \dots, v_n) OUT: $0 \neq v \in L(v_1, \dots, v_n)$ GOAL: minimize $\ l\ _1$
tests variables	$\Psi = (\psi_1, \dots, \psi_n)$ $\mathcal{V} = (x_1, \dots, x_m)$
lists of satisfying assignments	$\mathcal{R}_{\psi_1}, \dots, \mathcal{R}_{\psi_n}$, where $\mathcal{R}_{\psi_i} \subseteq \mathbf{F}^{\mathcal{V}_{\psi_i}}$ where \mathcal{V}_{ψ_i} is set of variables of ψ_i and \mathbf{F} a field.
super assignment S	function $S : \psi \mapsto S(\psi) \in \mathbf{Z}^{\mathcal{R}_{\psi}}$ for each $\psi \in \Psi$ $S(\psi) = (S(\psi)[r_1], \dots, S(\psi)[r_k])$ where $\mathcal{R}_{\psi} = \{r_1, \dots, r_k\}$
projection $S(\psi) _x$	$\forall a \in \mathbf{F}$ set $S(\psi) _x[a] := \sum_{r, r _x=a} S(\psi)[r]$ where x variable of ψ
S consistent	$\forall \psi, \phi \in \Psi, x \in \mathcal{V}$ holds $S(\psi) _x = S(\phi) _x$
S natural	$S(\psi) = (0, \dots, 0, 1, 0, \dots, 0)$ for each ψ
S non-trivial	$S(\psi) \neq (0, \dots, 0)$ for each ψ
norm $\ S\ $	$\ S\ := (1/ \Psi) \sum_i \ S(\psi_i)\ _1$
g-SSAT	IN: $(\Psi, \mathcal{V}, \mathcal{R}_{\psi_1}, \dots, \mathcal{R}_{\psi_n})$ OUT: distinguish YES: \exists consistent natural S NO: \forall consistent non-trivial S holds $\ S\ > g$.
g-CVP	IN: $v_1, \dots, v_n \in \mathbf{Z}^k, y \in \mathbf{R}^k, d \in \mathbf{R}$ OUT: distinguish YES: $\exists v \in L(v_1, \dots, v_n)$ s. t. $\ y - v\ _1 \leq d$ NO: $\forall v \in L(v_1, \dots, v_n)$ holds $\ y - v\ _1 > gd$.
g-SIS	IN: integer matrix $B = (b_1, \dots, b_n)$, target vector $t \in L(b_1, \dots, b_n)$ and $d \in \mathbf{R}$ OUT: distinguish YES: \exists solution $\sum_i a_i b_i = t$ with $\ b\ _1 \leq d$ NO: \forall solutions $\sum_i a_i b_i = t$ holds $\ b\ _1 > gd$

OUTLINE: g' -CVP $>$ g' -SIS $>$ g -SSAT ($>$ PCP) $>$ SAT where $g^{(l)} = n^{c^{(l)}/\log \log n}$ where $n = |\Psi|$ and $c^{(l)} > 0$ is some constant.

SAT[F] as a consistency problem

Input
 $\Phi = \psi_1, \dots, \psi_n$ Boolean functions - 'tests'
 x_1, \dots, x_m variables with range \mathbf{F}
 for each test: a list of satisfying assignments

Problem
 Is there an assignment *to the tests* that is consistent?

$\psi_1(x,y,z)$	$\psi_2(w,x,z)$	$\psi_3(y,w,x)$
(0,2,7)	(1,0,7)	(0,1,0)
(2,3,7)	(1,3,1)	(2,1,0)
(3,1,1)	(3,2,2)	(2,1,5)

Super-Assignments

A natural assignment for $\psi(x,y,z)$ $\psi(x,y,z)$'s super-assignment

$\|S(\psi)\| = |-2| + |2| + |3| = 7$ Norm S - Average $\|A(\psi)\|$ over Ψ

Consistency

In the SAT case:

$A(\psi) = (3, 2, 5)$
 $A(\psi)|_x := (3)$

$\forall x \forall \phi, \psi$ that depend on x : $A(\phi)|_x = A(\psi)|_x$

Consistency

$S(\psi) = +3(1, 1, 2) \oplus -2(3, 2, 5) \oplus 2(3, 3, 1)$

$S(\psi)|_x := +3(1) \oplus 0(3)$

Consistency: $\forall x \forall \phi, \psi$ that depend on x : $S(\phi)|_x = S(\psi)|_x$

g-SSAT - Definition

Input:
 $\Phi = \phi_1, \dots, \phi_n$ tests over variables x_1, \dots, x_m with range \mathbf{F}
 for each test ϕ_i - a list of sat. assign. \mathcal{R}_{ϕ_i}

Problem: Distinguish between
 [Yes] There is a natural assignment for Φ
 [No] Any non-trivial consistent super-assignment is of norm $> g$

Theorem: SSAT is NP-hard for $g = n^{c/\log \log n}$
 (conjecture: $g = n^\epsilon$, $\epsilon = \text{some constant}$)