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Primitive Shape Characterization using Interval Methods

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Introduction

Introduction

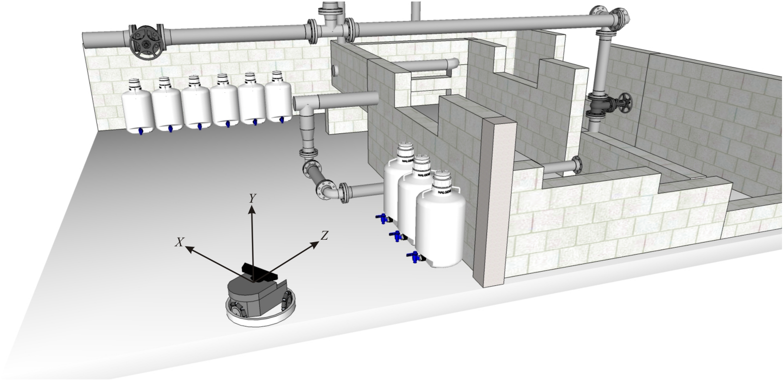
In the nuclear industry, the need for characterization of objects immerse in hazardous environments has become an urgent need.

Such objects correspond to nuclear equipment such as vessels, pipelines, containers, etc. located in closed shelters that are partially known.

The above is mainly due to the lack of blueprints or knowledge about the distribution of those objects.

The characterization and detection of those objects is required for tasks such as cutting, transportation and decommissioning.

Problem definition



The aim of this work consist in *the development of a system capable to identify and characterize objects using laser scanners mounted on mobile robots.*

Our Approach

We make use of the structured-light scanner Kinect to retrieve depth information in the context of a point cloud:

$$\mathbf{P} = (x_i, y_i, z_i),\ i = \{1 \ldots m\}$$

This type of sensor have measurement uncertainty defined by upper and lower bounds with no further assumption about its probability distribution [4].

Why interval methods?

Due to the nature of our data, interval analysis is suitable because:

The representation of the measurements as an interval is adequate to represent variables with uncertain probability density functions.

The characterization of the shapes is related by constrains that can be propagated in order to compute intervals that are guaranteed to contain all feasible values for the interest variables.

Why interval methods?

Due to the nature of our data, interval analysis is suitable because:

- The representation of the measurements as an interval is adequate to represent variables with uncertain probability density functions.
- The characterization of the shapes is related by constrains that can be propagated in order to compute intervals that are guaranteed to contain all feasible values for the interest variables.

Set Estimation

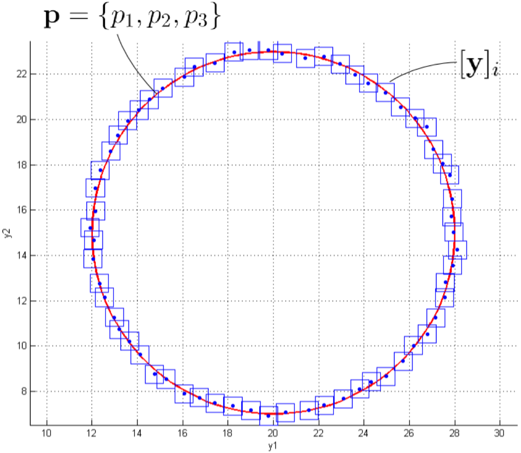
Set Estimation

The detection of circular shapes on images using interval methods has been investigated by Jaulin [2], as a problem of parameter estimation under error-bounded estimation basis [3]. This is, the characterization of a set defined as follows:

$$\mathbb{P} = \bigcap_{i \in \{1,\ldots,m\}} \underbrace{\{\mathbf{p} \in \mathbb{R}^{n_p}, \exists [\mathbf{y}] \in [\mathbf{y}]_i, \mathbf{f}(\mathbf{p},\mathbf{y}) = 0\}}_{\mathbb{P}_i} \quad (1)$$

being $\mathbf{p}$ the parameter vector, $[\mathbf{y}]_i \subset \mathbb{R}^{n_y}$ is the ith measurement box and $\mathbf{f}$ is the model function. In this sense, the set $\mathbb{P}_i$ is the set of all parameters vector consistent with the ith measurement box.

Set Estimation



Shape Extraction as Set Estimation

Shape Extracion

The shape extraction can be defined as a set estimation problem [2], thus

$$\mathbf{f}:\begin{cases}\mathbb{R}^{n_p}\times\mathbb{R}^{d}\rightarrow\mathbb{R}^{n_f}\\ (\mathbf{p},\mathbf{y})\qquad\rightarrow\mathbf{f}(\mathbf{p},\mathbf{y})\end{cases}\tag{2}$$

being $d\in\{2,3\}$ the dimension of the analyzed shape. The vector $\mathbf{y}\in\mathbb{R}^d$ corresponds to a point in the cloud and $\mathbf{p}$ is the parameter vector that corresponds to the shape under analysis.

Shape Extracion

Under this basis, the shape that corresponds to the vector $\mathbf{p}$ is defined as follows:

$$\mathcal{S}(\mathbf{p}) \stackrel{\text{def}}{=} \left\{ \mathbf{y} \in \mathbb{R}^d, \mathbf{f}(\mathbf{p}, \mathbf{y}) = 0 \right\} \tag{3}$$

Taking into consideration a set of boxes (measurements) in the primitive shape space dimension d, each of this boxes is assumed to touch the periphery of the considered shape.

Shape Extracion

In this sense, the concepts presented in [2], [3] for circles, can be extended to a 3-dimensional space. In our particular case for:

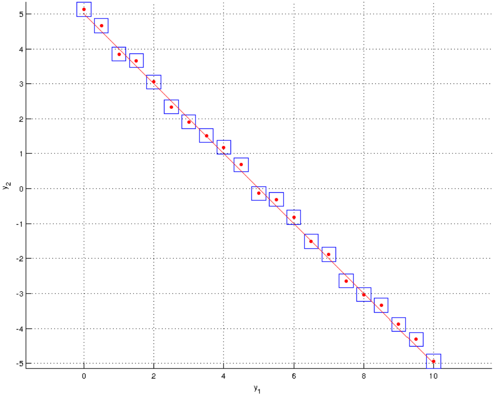
- Planes
- Spheres
- Cylinders

Results

Results: 2D Line

In a 2D context, the model function of a line can be defined as:

$$\mathbf{f}(\mathbf{p}, \mathbf{y}) = p_1 y_1 - y_2 + p_2 \tag{4}$$



Results: 2D Line

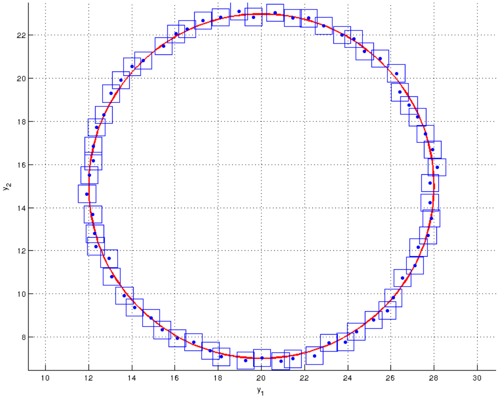
Parameter	Initial box	Parameters Detected	Real Value
p_1	$[-100, 100]$	$[-1{,}05765, -0{,}933176]$	-1
p_2	$[-100, 100]$	$[4{,}74225, 5{,}31999]$	5

Table 1: Parameters of the line.

Results: 2D Circle

The model function of a circle can be defined as:

$$\mathbf{f}(\mathbf{p}, \mathbf{y}) = (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2 \quad (5)$$



Results: 2D Circle

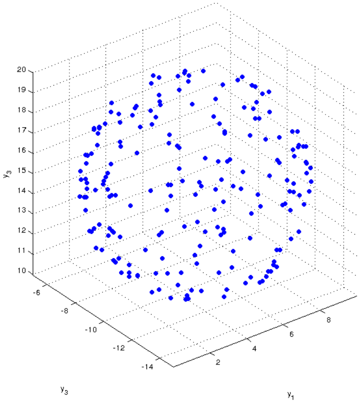
Parameter	Initial box	Parameters Detected	Real Value
p_1	$[-100, 100]$	$[19{,}883, 20{,}2792]$	20
p_2	$[-100, 100]$	$[14{,}7971, 15{,}2025]$	15
p_3	$[0, 100]$	$[7{,}75497, 8{,}15122]$	8

Table 2: Parameters of the circle.

Results: Sphere

The model function of a sphere can be defined as:

$$\mathbf{f}(\mathbf{p}, \mathbf{y}) = (y_1 - p_1)^2 + (y_2 - p_2)^2 + (y_3 - p_3)^2 - p_4^2 \quad (6)$$



Results: Sphere

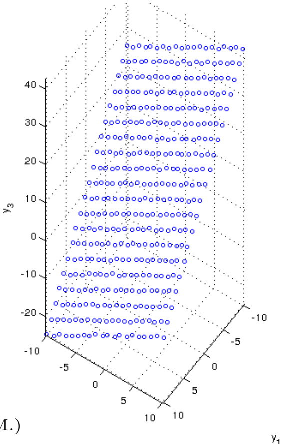
Parameter	Initial box	Parameters Detected	Real Value
p_1	$[-100, 100]$	$[4{,}68662, 5{,}28422]$	5
p_2	$[-100, 100]$	$[-10{,}2244, -9{,}62335]$	-10
p_3	$[-100, 100]$	$[14{,}6709, 15{,}2735]$	15
p_4	$[0, 100]$	$[4{,}71832, 5{,}30302]$	5

Table 3: Parameters of the sphere.

Results: Plane

The model function of a plane can be defined as:

$$\mathbf{f}(\mathbf{p}, \mathbf{y}) = y_1 p_1 - y_2 + y_3 p_2 + p_3 \tag{7}$$



Results: Plane

Parameter	Initial box	Parameters Detected	Real Value
p_1	$[-300, 300]$	$[2{,}75325, 3{,}10261]$	$2{,}9444$
p_2	$[-300, 300]$	$[1{,}09225, 1{,}23066]$	$1{,}1667$
p_3	$[-300, 300]$	$[-10{,}9862, -9{,}54395]$	$-10{,}3889$

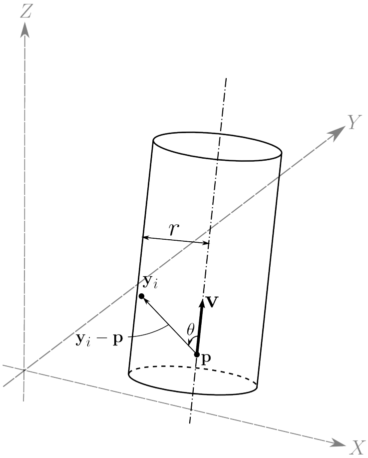
Table 4: Parameters of the plane.

Results: Cylinder

A circular cylinder can be defined by a radius r, a vector $\mathbf{v} = \{v_x, v_y, v_z\}$ that defines its central axis and a pivot point $\mathbf{p} = (p_x, p_y, p_z)$ lying on the axis.

Using some geometrical relations between the measurements $(\mathbf{y}_i)$ and the parameters that define the cylinder is possible to write an equation that describes the cylinder.

Results: Cylinder



$$r = \frac{||\mathbf{v} \times (\mathbf{y}_i - \mathbf{p})||}{||\mathbf{v}||} \quad (8)$$

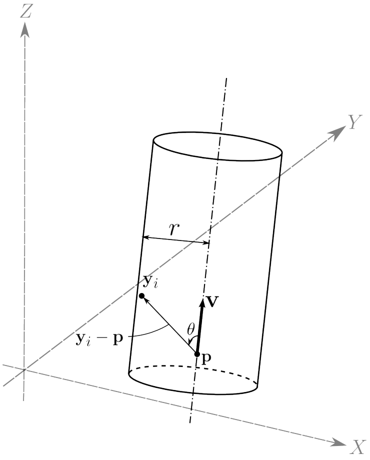
$$\mathbf{v} \cdot \mathbf{p} = 0 \quad (9)$$

This results in the problem of determining the parameter set:

$$\mathbf{p} = \{v_x, v_y, v_z, p_x, p_y, p_z, r\}$$

which is highly computationally expensive.

Results: Cylinder



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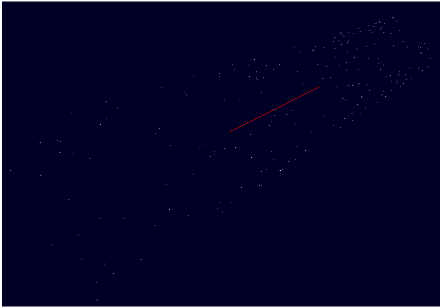
Results: Cylinder

To solve this issue, a methodology to convert the cylinder shape detection problem into a circle detection in two dimensions is proposed:

Results: Cylinder

a) Principal component analysis to detect a first guess of the central axis

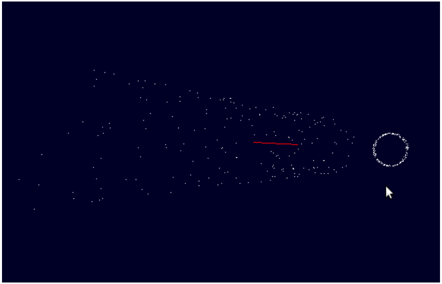


Results: Cylinder

b) The point cloud is projected on the plane whose normal is defined by the principal component.

The projected point cloud is analyzed using IM to detect a circle.

If an empty set is detected, the principal component is rotated using small steps until a circle is detected or the search space is exhausted.



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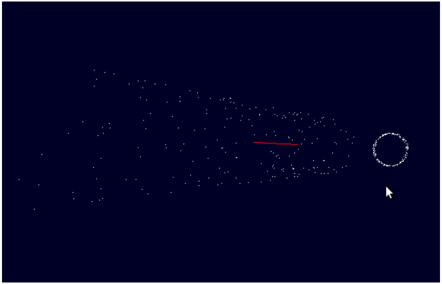

Results: Cylinder

Parameter	Value
v_x	$0{,}21984$
v_y	$-0{,}860446$
v_z	$-0{,}459678$
p_x	$[12{,}9335, 13{,}4998]$
p_y	$[5{,}4344, 5{,}4829]$
p_z	$[-4{,}0778, -3{,}7162]$
r	$[2{,}82329, 3{,}18471]$

Table 5: Parameters of the cylinder.

Conclusions

Conclusions

The characterization of 3-dimensional primitives shapes has been achieved using interval methods.

Interval methods is a tool suitable for applications on which the statistical distribution is unknown, which is the case in this work. The interval methods analysis was implemented in C++ and using Ibex [1].

Conclusions

In order to make the system robust to outliers the implementation of Q-intersection is mandatory.

An evaluation in therms of robustness, computational time, and accuracy against traditional methods will be held.

References

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