Interval Methods for Mobile Robot Mapping

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Outline

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SLAM Problem

- SLAM stands for Simultaneous Localization And Mapping. It means that a mobile robot needs to explore unknonwn environment while building a map and localizing itself within such map.
- If the robot knows the map of the environment and it detects familiar landmarks, localization is easy (Localization problem).
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- SLAM parameters (robot pose and landmarks locations in the map) can be estimated using two models:
  - Motion model that estimates the robot pose using proprioceptive sensor,e.g., encoder or IMU.
  - Observation model that estimates the landmark loaction using the exteroceptive sensor, e.g., LIDAR or Camera.
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Convergence of Different SLAM Approaches

- For SLAM, Building an accurate map leads to an accurate localization.
- Extended Kalman Filter (EKF) SLAM and FastSLAM (Particle Filter) approachs converge to the real map if:
  - Motion model and Observation model are linear.
  - Uncertainty in the motion model and the observation model are Gaussians.
  - The location of one landmark is known in advance.
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Constraint Satisfaction Problem (CSP).

- Consider the system of m equations with n variables, such that: $f_j(x_1, x_2, ..., x_n) = 0, \;\; j = 1 : m$, where $x_i \in [x_i]$, $[\mathbf{x}] = [x_1] \times [x_2] \times ... \times [x_n]$, and $\mathbf{f}(\mathbf{x}) = \mathbf{0}$.
- A *constraint satisfaction problem* (CSP) $\mathcal{H}$, is defined as:

$$\mathcal{H} : (\mathbf{f}(\mathbf{x}) = \mathbf{0}, \;\; \mathbf{x} \in [\mathbf{x}])$$

- The *solution set* of $\mathcal{H}$ is defined as:

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Definition of Contractors.

- Contracting $\mathcal{H}$ means replacing $[\mathbf{x}]$ by a smaller domain $[\mathbf{x}']$ such that the solution set remains unchanged, i.e., $\mathbb{S} \subset [\mathbf{x}'] \subset [\mathbf{x}]$.
- A *contractor* $\mathcal{C}$ for $\mathcal{H}$ is an operator that compute the subset $[\mathbf{x}']$, and it is defined formally as follows: **Definition:** A contractor $\mathcal{C}$ is a mapping from $\mathbb{IR}^n$ to $\mathbb{IR}^n$ such that:

$$\begin{aligned} \forall[\mathbf{x}] \in \mathbb{IR}^n, \quad & \mathcal{C}([\mathbf{x}]) \subset [\mathbf{x}] && (contractance) \\ & \mathcal{C}([\mathbf{x}]) \cap \mathbb{S} = [\mathbf{x}] \cap \mathbb{S} && (correctness) \end{aligned}$$

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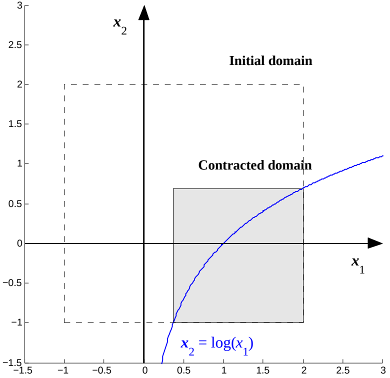
Forward-Backward Propagation Contractor.



- Contractor $\mathcal{C}$ applied to $[\mathbf{x}] = [-1, 2] \times [-1, 2]$ and results in $[\mathbf{x}'] = [0.3679, 2] \times [-1, 0.6931]$.

Outline

Problem Statement

- Consider a robot moving in an unknown environment with motion model defined by equation (1), where $\mathbf{s}_k$ and $\mathbf{u}_k$ are the robot pose and the control inputs at time k, respectively.

$$\mathbf{s}_{k+1} = \mathbf{f}(\mathbf{s}_k, \mathbf{u}_k) \quad (1)$$

- The robot can detect static landmarks (assuming data association is solved) in the environment using the observation model defined by equation (2), where, $\mathbf{m}_i$ is the location of the i^{th} landmark in the environment.

$$\mathbf{z}_{k,i} = \mathbf{g}(\mathbf{s}_k, \mathbf{m}_i) \quad (2)$$

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Problem Statement

- Since the robot sensors are noisy, their uncertainties are assumed to be bounded such that:

$$\mathbf{s}_{k+1} - \mathbf{f}(\mathbf{s}_k, \mathbf{u}_k) \in [\omega_k] \quad (3)$$

$$\mathbf{z}_{k,i} - \mathbf{g}(\mathbf{s}_k, \mathbf{m}_i) \in [\nu_{k,i}] \quad (4)$$

- The goal is to estimate all robot poses for all time instances $k \in \{0, ..., k_{max}\}$, and all locations of landmarks that are consistent with all control inputs and all observations.
- All the parameters to be estimated are represented by the vector $\mathbf{x}$ in equation (5):

$$\mathbf{x} = [\mathbf{s}_0, ..., \mathbf{s}_{k_{max}}, \mathbf{m}_1, ..., \mathbf{m}_M]^T \quad (5)$$

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Parameters Estimation

- Equations (3-5) represent a constrait satisfaction problem (CSP).
- Contractors $C_k^{\mathbf{s}}$ and $C_{k,i}^{\mathbf{s},\mathbf{m}}$ are used for CSP such that:

$$\mathbf{s}_{k+1} - \mathbf{f}(\mathbf{s}_k, \mathbf{u}_k) \in [\omega_k] \rightarrow C_k^{\mathbf{s}} \quad (6)$$

$$\mathbf{z}_{k,i} - \mathbf{g}(\mathbf{s}_k, \mathbf{m}_i) \in [\nu_{k,i}] \rightarrow C_{k,i}^{\mathbf{s},\mathbf{m}} \quad (7)$$

- From an initial box $[\mathbf{x}]$, the following contractor is defined to compute the enclosure of the SLAM solution:

$$C^{\mathbf{x}} = \left(\bigcap_{k \in \{0,\ldots,k_{max}\}} \left(C_k^{\mathbf{s}} \circ \bigcap_i C_{k,i}^{\mathbf{s},\mathbf{m}} \right) \right)^{\infty} \quad (8)$$

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1-D: Assumptions

- Observation model is in the form of:

$$\mathbf{z}_{k,i} = \mathbf{g}(\mathbf{s}_k, \mathbf{m}_i) \Rightarrow z_{k,i} = g(m_i - s_k) \tag{9}$$

  where, g is any one-to-one nonlinear function.
- The observation model has bounded uncertainty such that:

$$z_{k,i} - g(m_i - s_k) \in [\nu_{k,i}] \tag{10}$$

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- At each time step k, the robot observes, at least, one *old* landmark.

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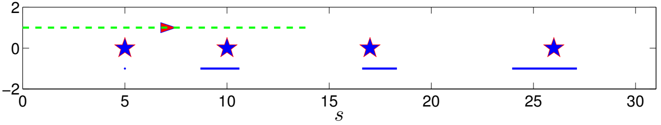
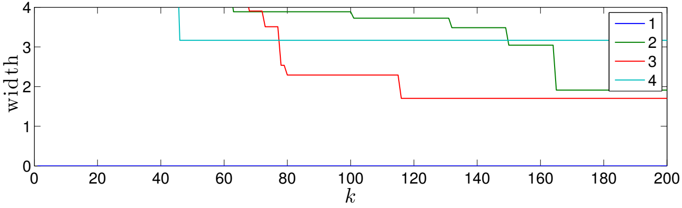
1-D: Results

$$g_{k,i}(s_k, m_i) = \sqrt{m_i - s_k} \quad , \quad [\nu_{i,k}] = [-0.5, 0.5]$$





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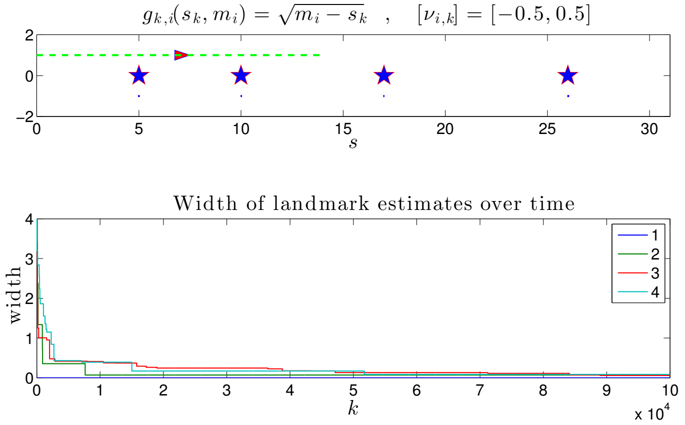
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2
0
−2
0
5
10
15
20
25
30
s
Width of landmark estimates over time
width
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
k
x 10^4
1
2
3
4

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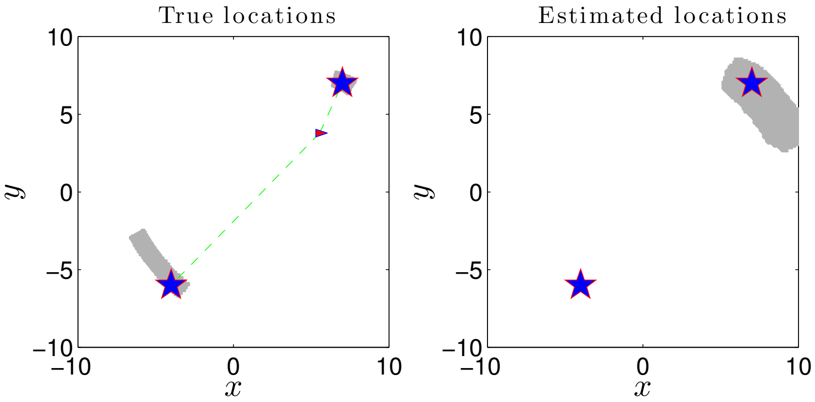

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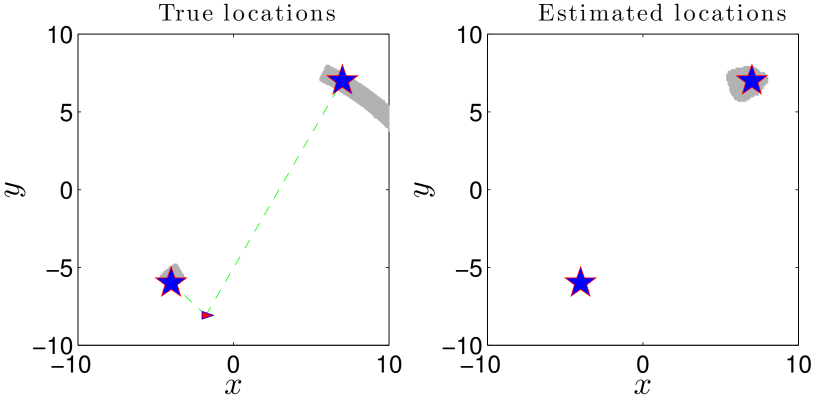

$k = 10$

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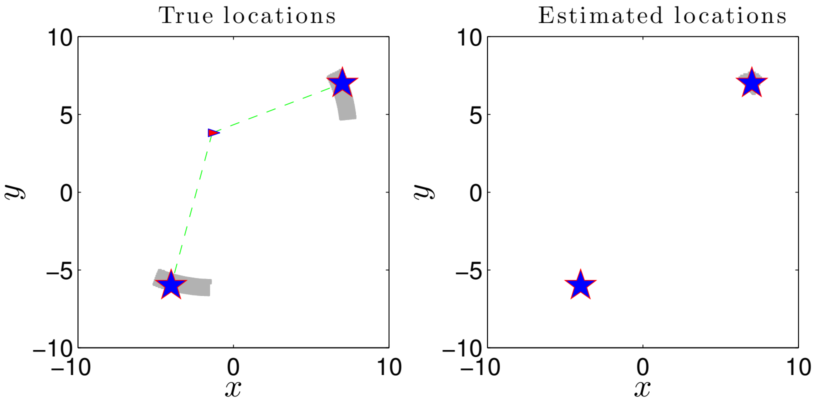

$k = 100$

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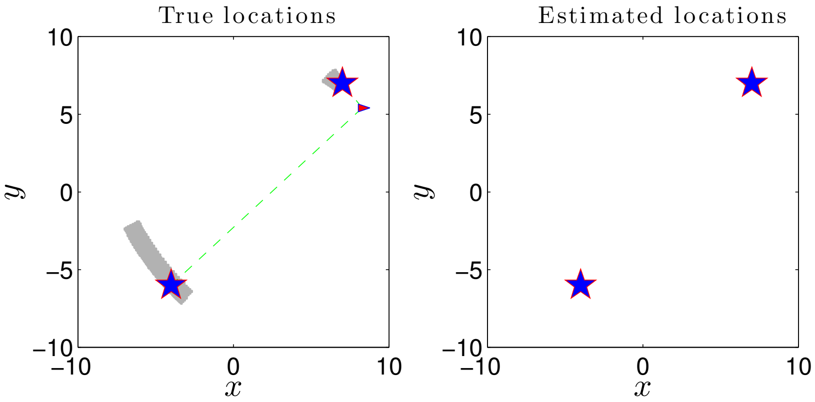

$k = 200$

Outline

1. Motivation
   - SLAM Problem
   - Why Mapping using Interval Methods
2. Mobile Robot Mapping using Interval Methods
   - Problem Statement
   - Parameters Estimation
3. Applications
   - Robot moving in 1-D Environment
   - Robot moving in 2-D Environment without Rotation
   - Robot moving in 2-D Environment with Rotation
4. Discussion

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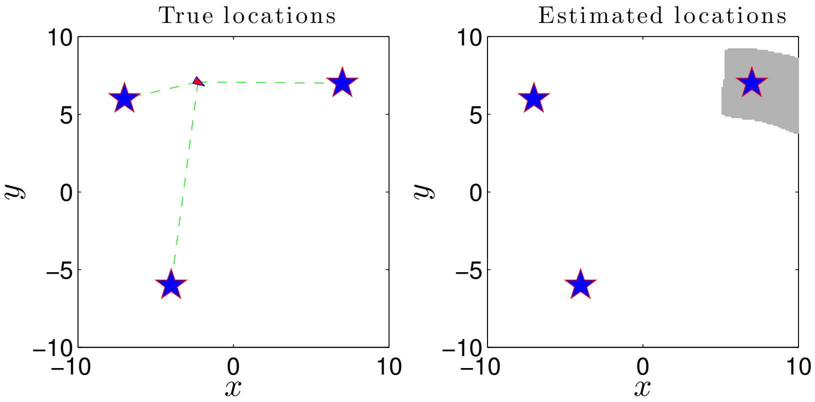
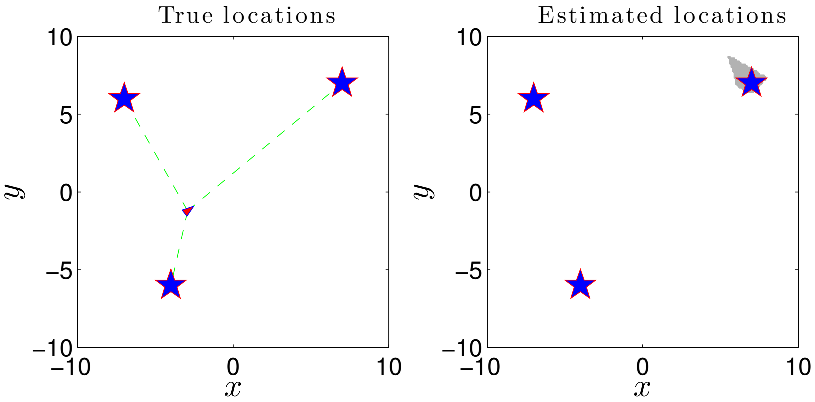
2-D without Rotation: Results



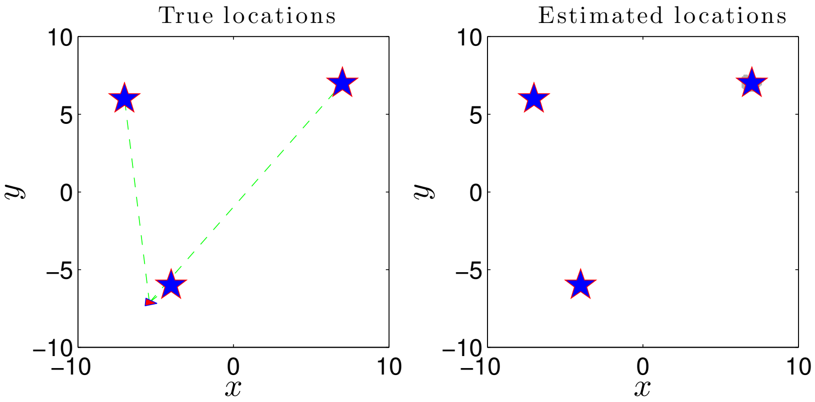
$k = 1$

2-D without Rotation: Results



$k = 10$

2-D without Rotation: Results



$k = 100$

Experimental Observations

- In the 1-D case, it is possible to proof that mapping using interval methods approach converges to the true map given the following conditions:
  - At least one landmark location is known in advance.
  - At any time instance k, at least one old landmark is observed.
  - The observation model is in the form of $z_{i,k} - g(m_i - s_k) \in [\nu_{i,k}]$, where g is a one-to-one nonlinear function, and the sensor noise is bounded in the form of interval, i.e. $[\nu_{i,k}]$.

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- **Proposition 2**: Let $[x] \in \mathbb{IR}$, if $x^{\star} \in [x]$, then, $0 \in [x] - x^{\star}$.
- **Theorem 1**: Let $0 \in [\nu_{i,k}]$, if g is one-to-one function, then:
  $$\bigcap_{k=0}^{\infty} \left([g^{-1}]\left(z_{i,k} - [\nu_{i,k}]\right) - [g^{-1}]\left(z_{j,k} - [\nu_{j,k}]\right) \right) = \{d_{i,j}\}$$
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Questions...