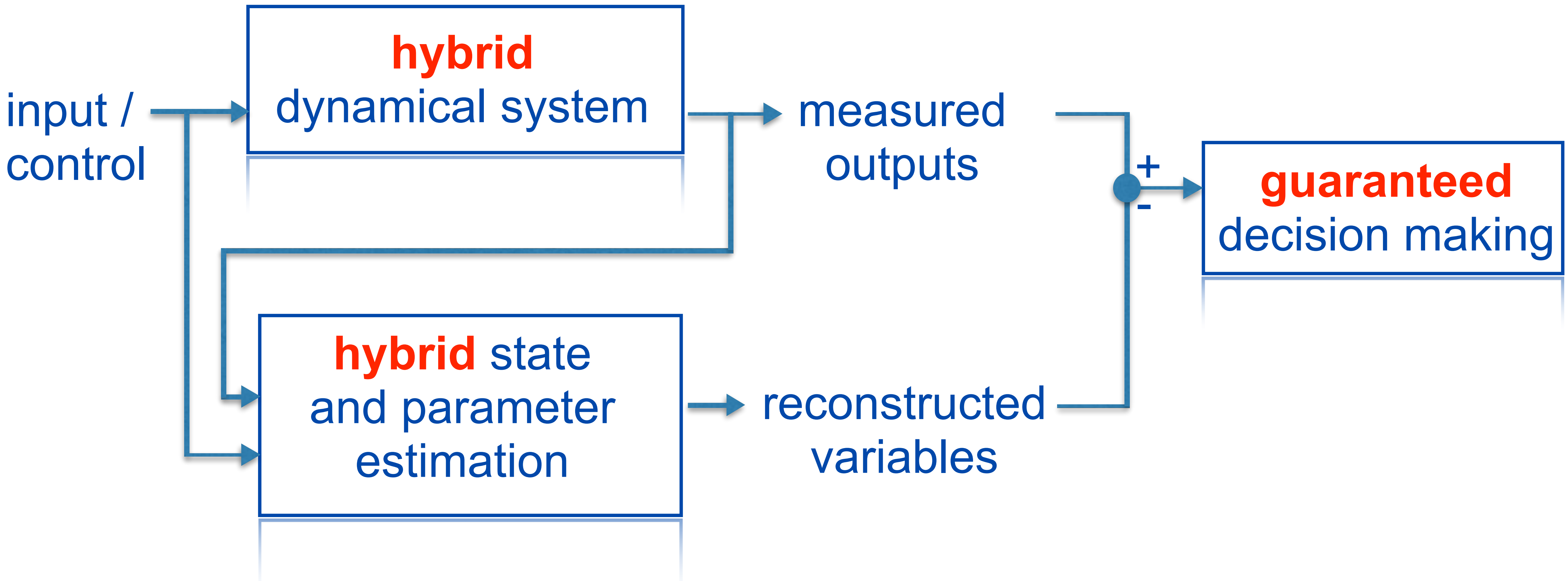
Parameter identification with hybrid systems in a bounded-error framework

Moussa MAIGA, Nacim RAMDANI, & Louise TRAVE-MASSUYES
Université d'Orléans, Bourges, and LAAS CNRS Toulouse, France.

SWIM 2015, Praha
9-11 June 2015

Model-based FDI

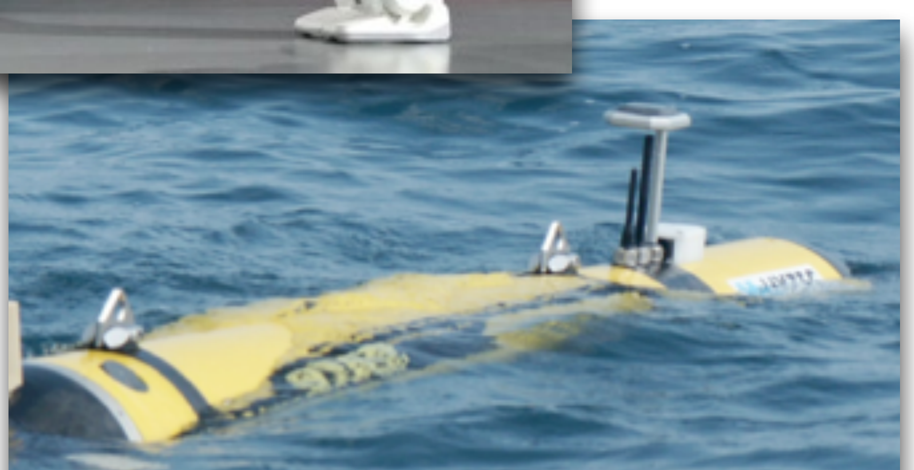


- Extend to hybrid dynamical systems set-membership approaches for model-based FDI

Outline

- **Hybrid dynamical systems**
- Set membership estimation
- Hybrid reachability approach
- Example
- Research directions

Hybrid Cyber-Physical Systems

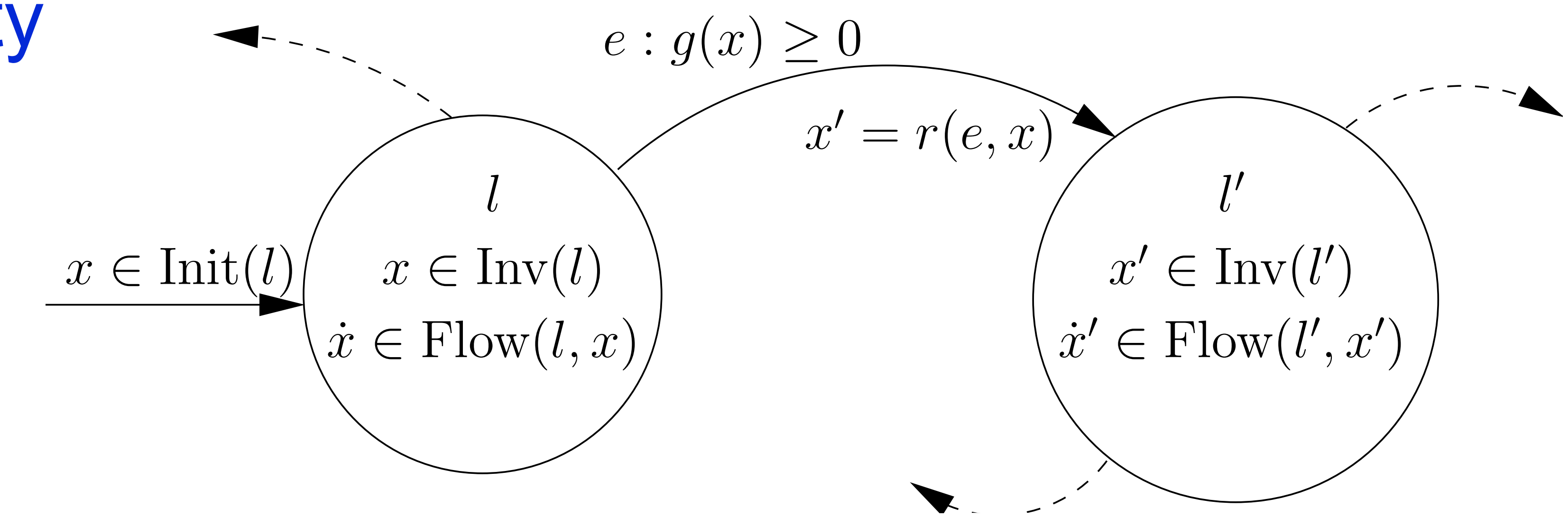


- Interaction discrete
  + continuous dynamics
- Safety-critical
  embedded systems
- Networked
  autonomous systems

Hybrid Cyber-Physical Systems

Modelling → hybrid automaton (Alur, et al. 1995)

- Non-linear continuous dynamics
- Bounded uncertainty



$$H = (\mathcal{Q}, \mathcal{D}, \mathcal{P}, \Sigma, \mathcal{A}, \mathsf{Inv}, \mathcal{F}),$$

Continuous dynamics

$$\begin{aligned} \mathsf{flow}(q) :&\quad \dot{\mathbf{x}}(t) = f_q(\mathbf{x}, \mathbf{p}, t), \\ \mathsf{Inv}(q) :&\quad \nu_q(\mathbf{x}(t), \mathbf{p}, t) < 0, \end{aligned}$$

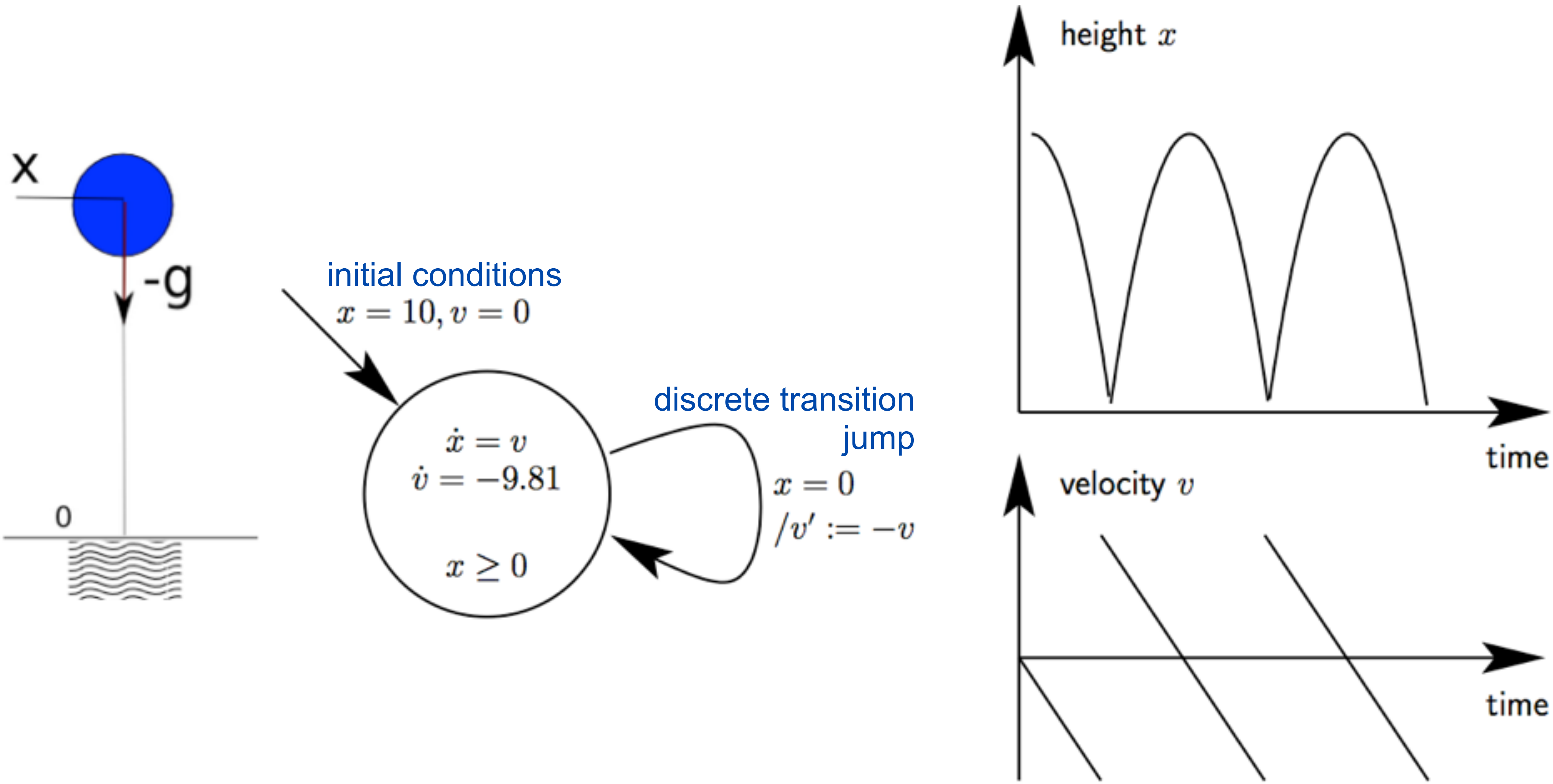
Discrete dynamics

$$\begin{aligned} \mathcal{A} \ni e :&\quad (q \to q') = (q, \mathsf{guard}, \sigma, \rho, q'), \\ \mathsf{guard}(e) :&\quad \gamma_e(\mathbf{x}(t), \mathbf{p}, t) = 0, \end{aligned}$$

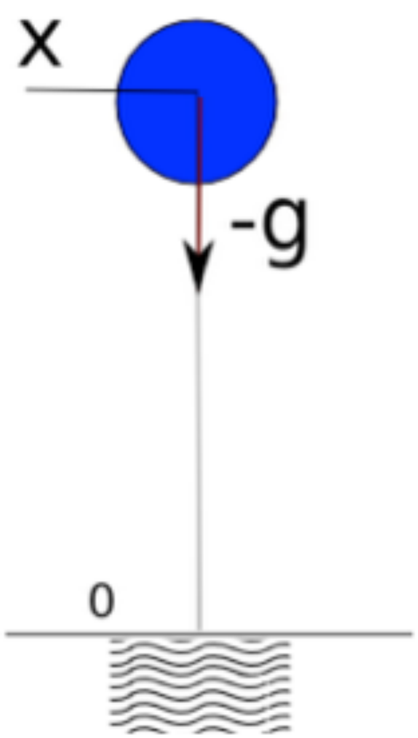
$$t_0 \leq t \leq t_N, \quad \mathbf{x}(t_0) \in \mathbb{X}_0 \subseteq \mathbb{R}^n, \quad \mathbf{p} \in \mathbb{P}$$

Hybrid Cyber-Physical Systems

■ **Example :** the bouncing ball

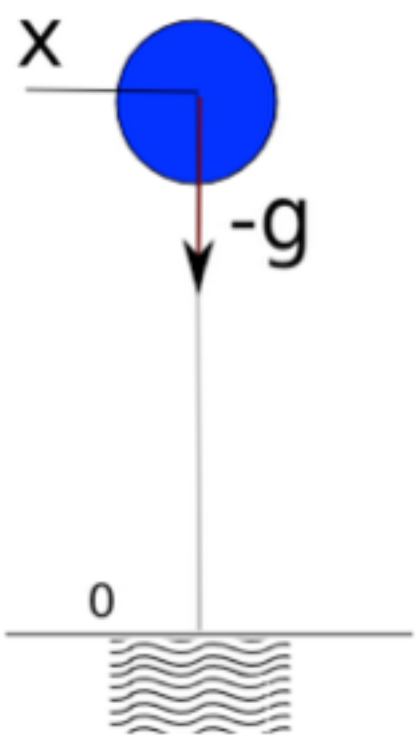


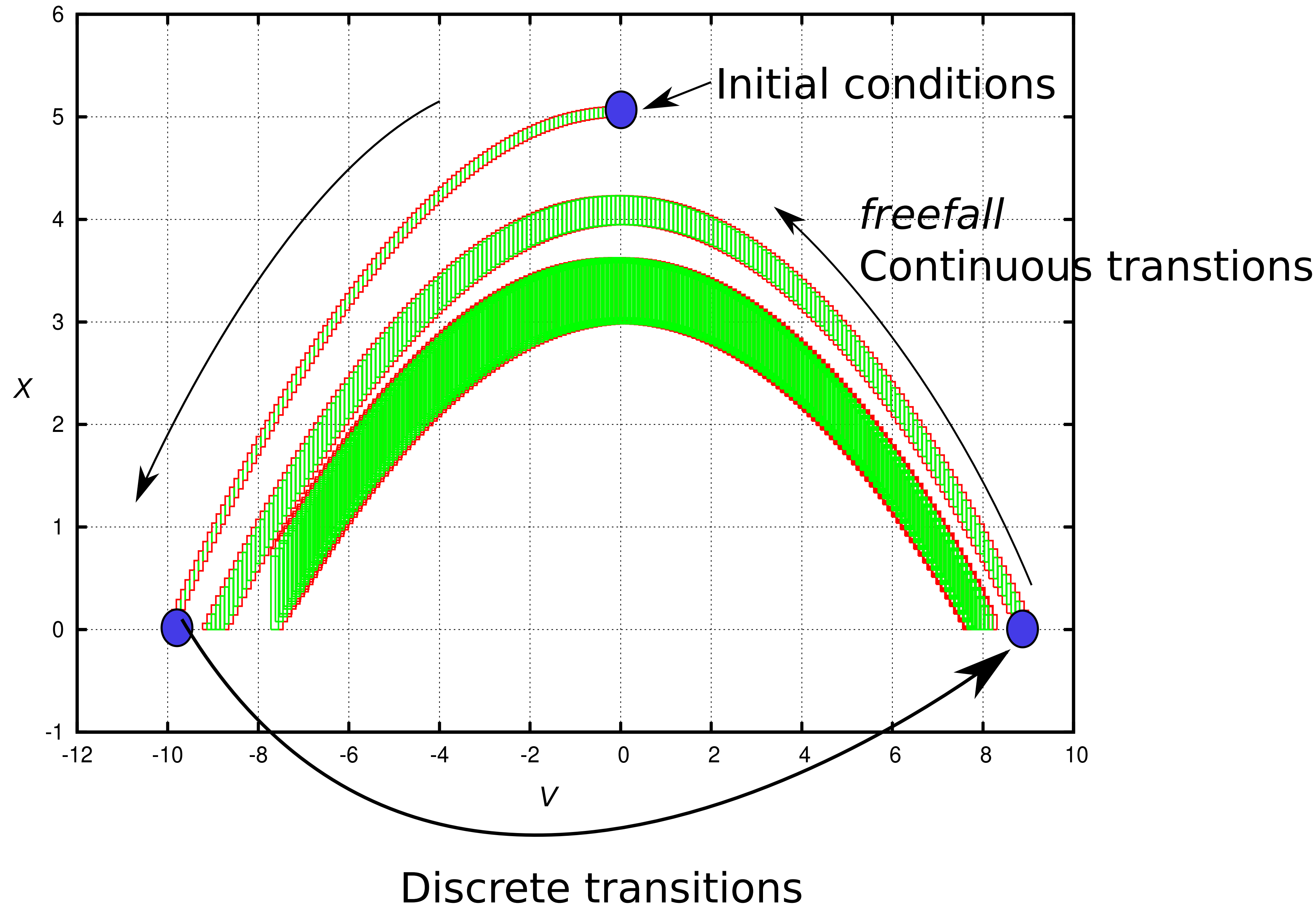
- **Example :** the bouncing ball



Hybrid Cyber-Physical Systems

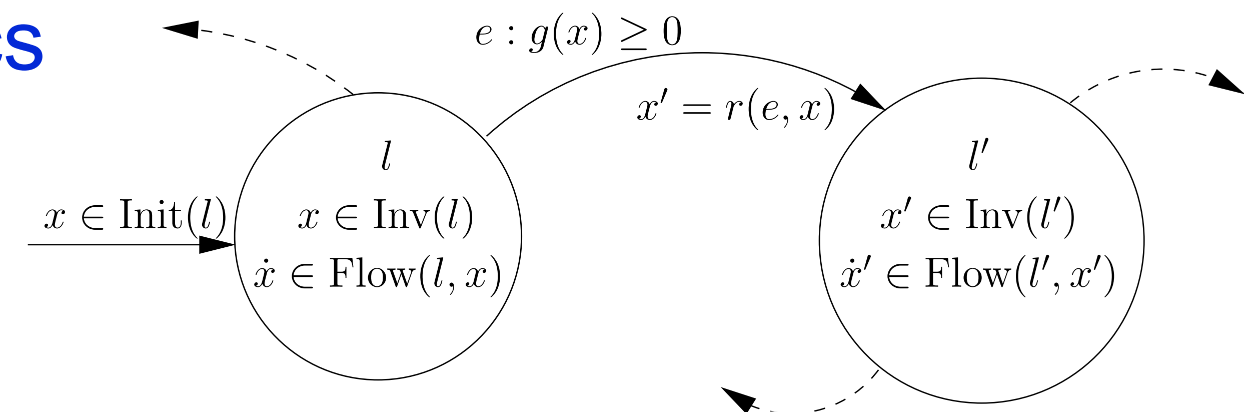
Example : the bouncing ball





Estimation of Hybrid State

- **Modelling** → **hybrid automaton**
  - Nonlinear continuous dynamics
  - Nonlinear guards sets
  - Nonlinear reset functions

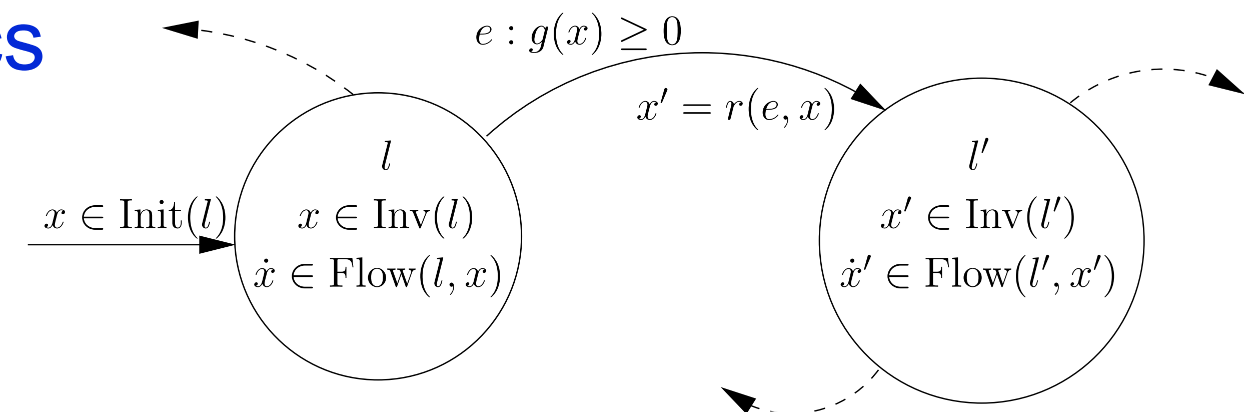


  - **Bounded** uncertainty

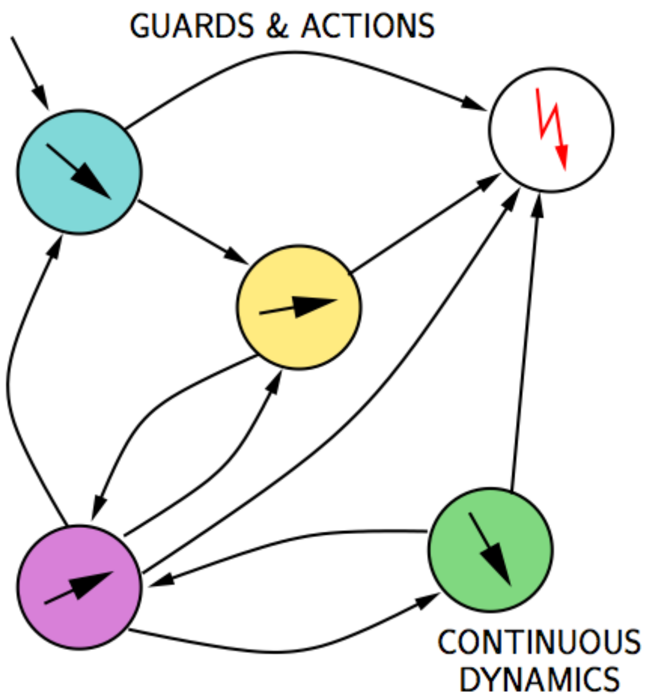
Estimation of Hybrid State

Modelling → hybrid automaton

- Nonlinear continuous dynamics
- Nonlinear guards sets
- Nonlinear reset functions



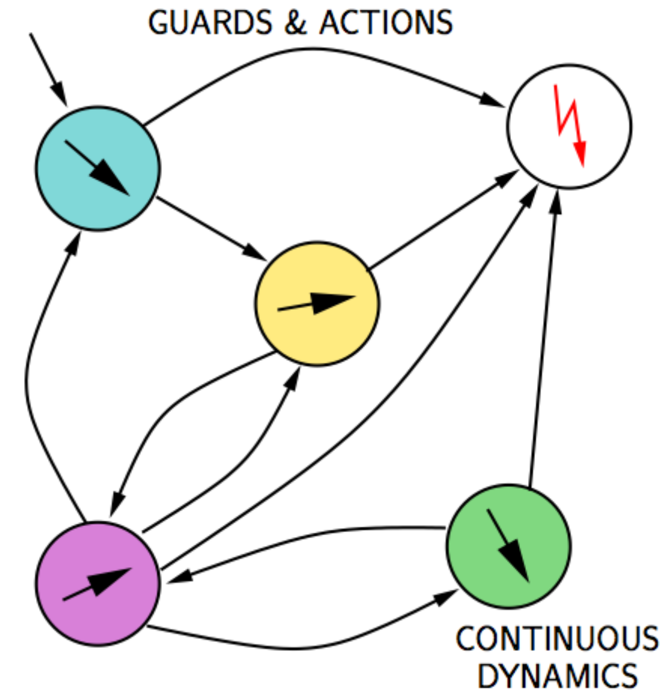
- **Bounded** uncertainty



Faults as discrete modes !!

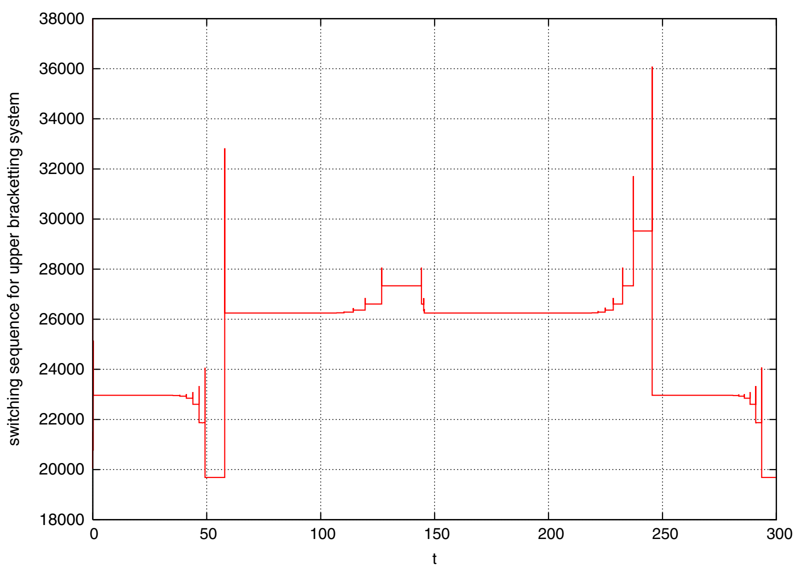
Estimation of Hybrid State

- **Modelling → hybrid automaton**
  - Nonlinear …
  - Bounded uncertainty



- **Faults as a mode !!**

- **FDI → State Estimation**
  → reconstruct system variables
  - switching sequence
  - continuous variables

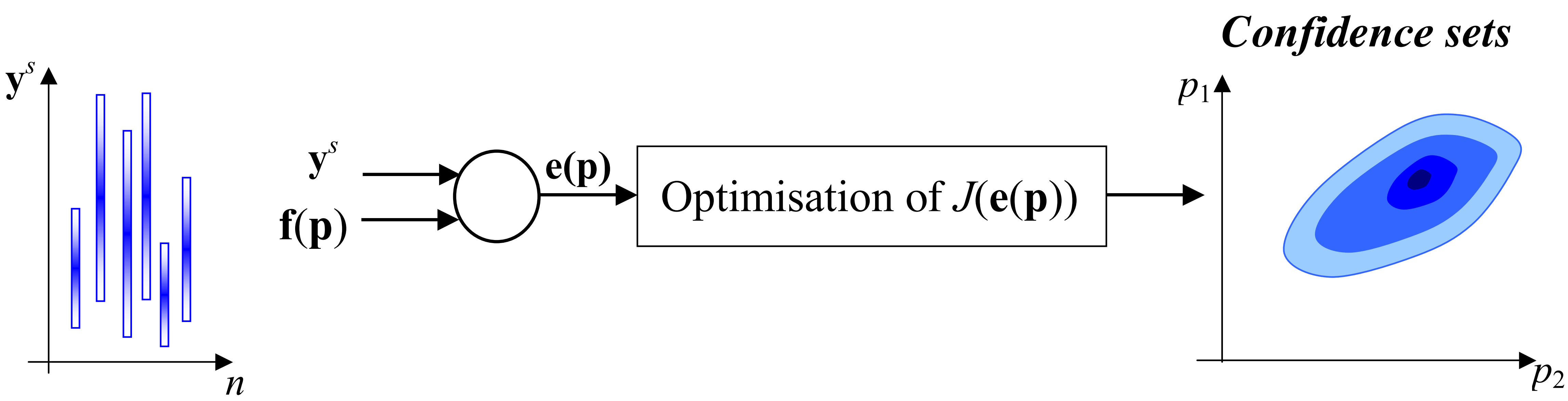


Outline

- Hybrid dynamical systems
- **Set membership estimation**
- Hybrid reachability approach
- Example
- Research directions

Classical Estimation

- Classical estimation is probabilistic

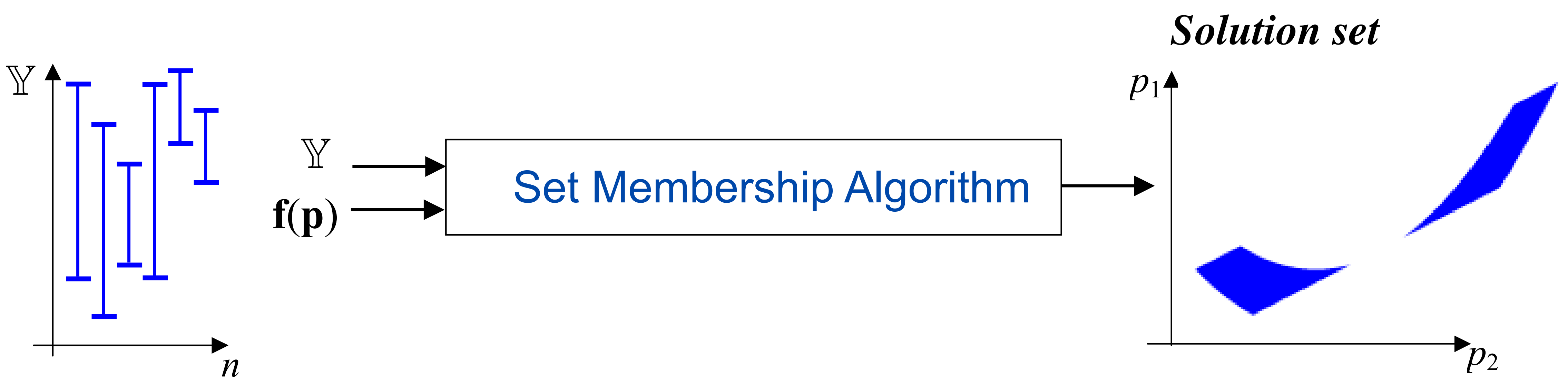


Yield valid results only if

- Perturbations, errors and model uncertainties with statistical properties known *a priori*
- Model structure is correct, no modeling errors

Set Membership Estimation

- Unknown but bounded-error framework



Hypothesis

Uncertainties and errors are bounded with known prior bounds

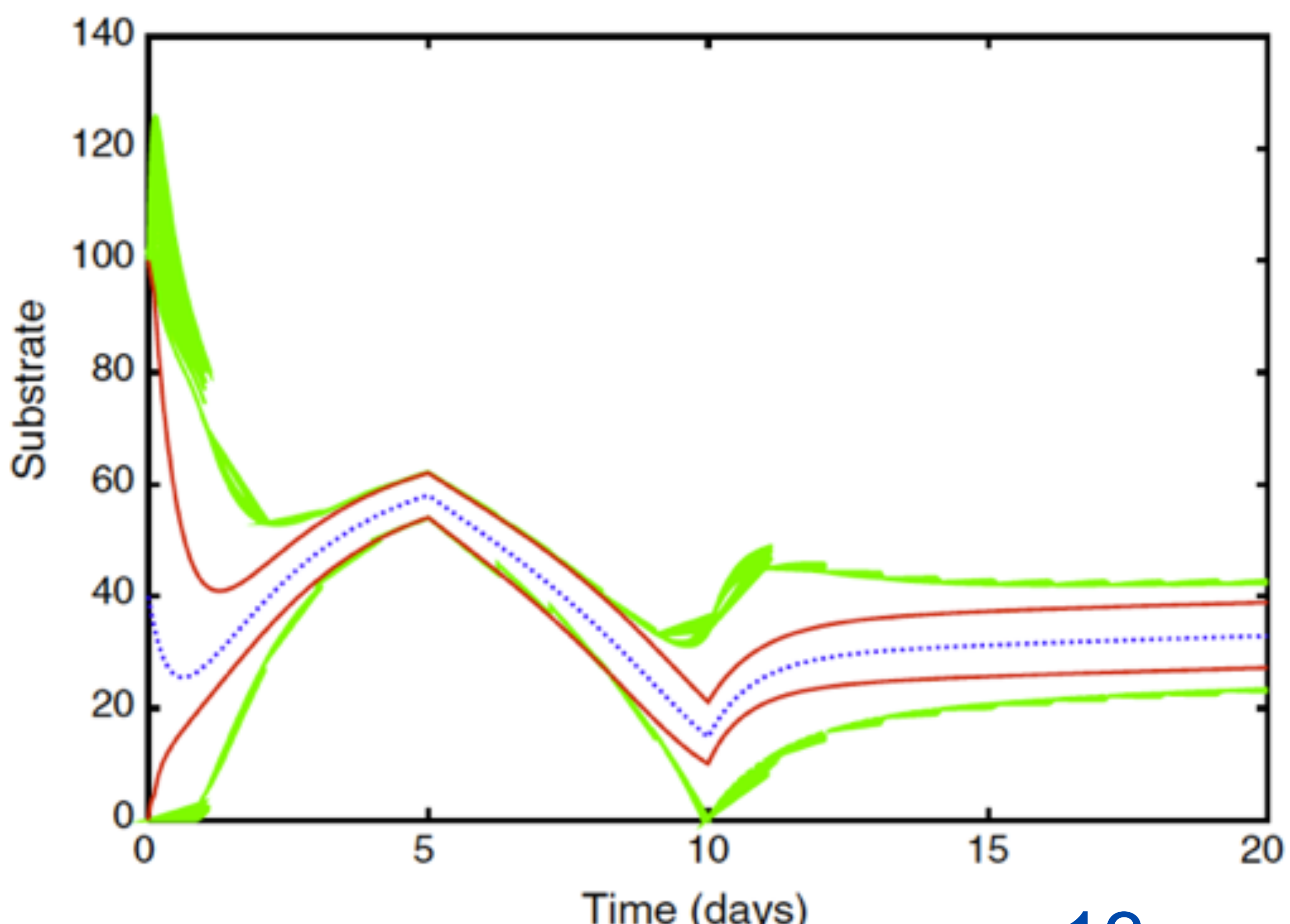
A set of feasible solutions

$$\mathbb{S} = \{\mathbf{p} \in \mathbb{P} | \mathbf{f}(\mathbf{p}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}) \cap \mathbb{P}$$

Set Membership Estimation

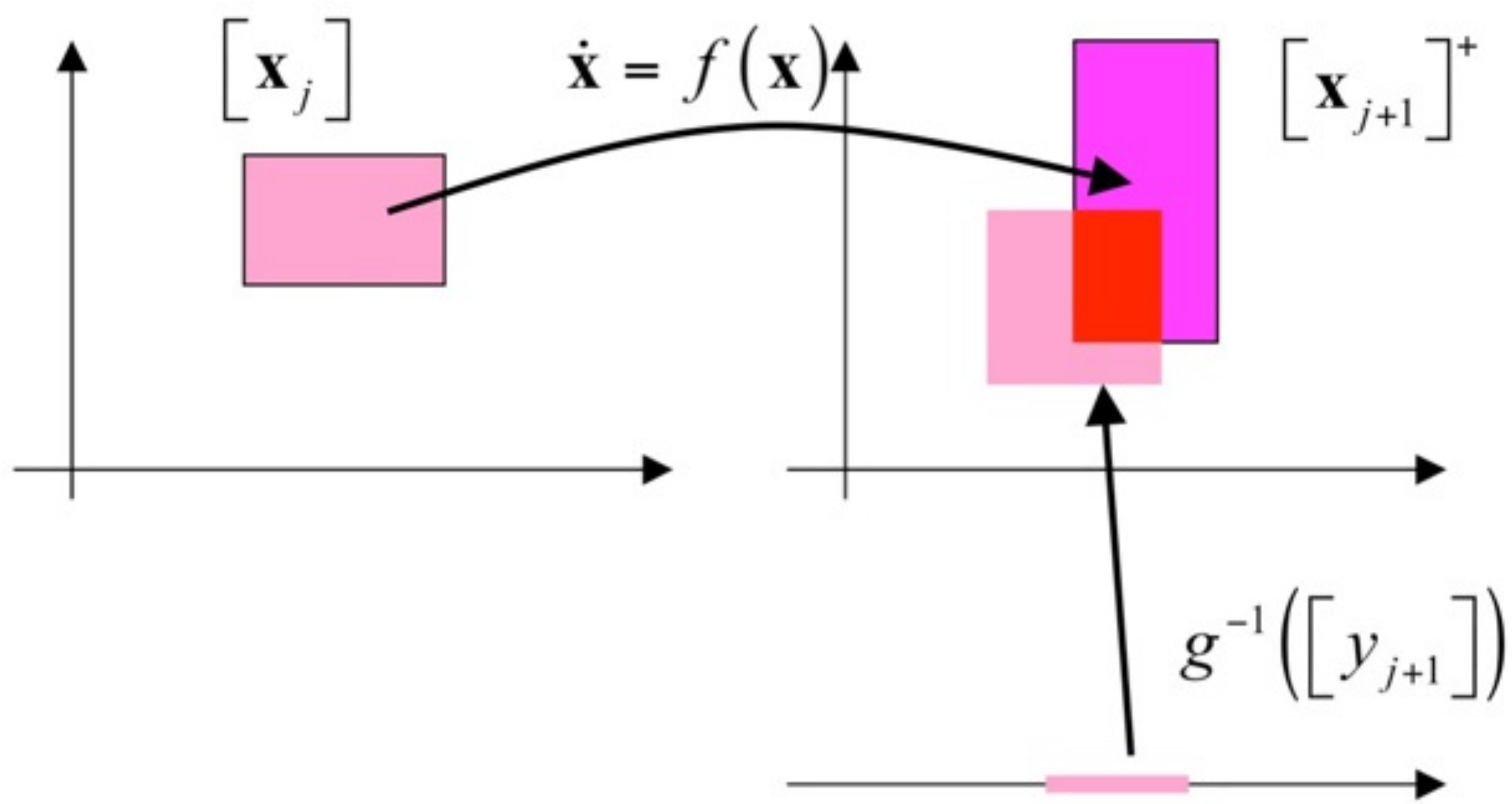
State estimation with continuous systems

- Interval observers
  - (Moisan, et al. 2009), (Meslem & Ramdani, 2011), (Raïssi, et al., 2012), (El Thabet, et al. 2014) ….



Set Membership Estimation

- **State estimation with continuous systems**
  - Prediction - Correction / Filtering approaches
    - (Raïssi et al., 2005), (Meslem, et al, 2010), (Milanese & Novara, 2011), (Kieffer & Walter, 2011) …



Set Membership Estimation

Set inversion. Parameter estimation

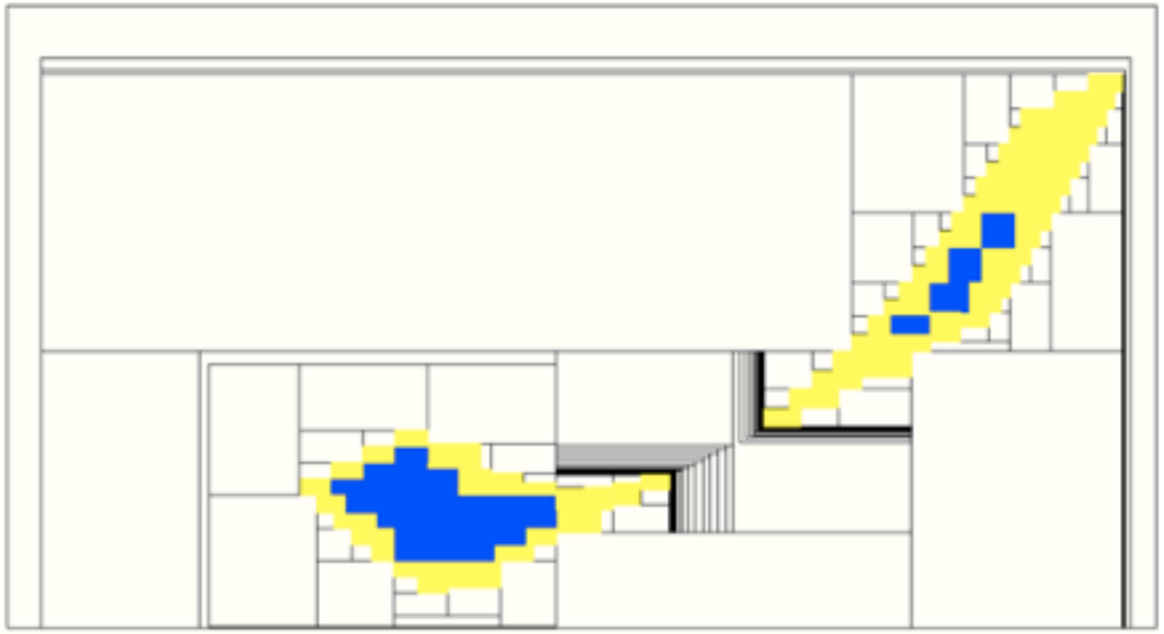
- Branch-&-bound, branch-&-prune, interval contractors …
  (Jaulin, et al. 93) (Raïssi et al., 2004)

$$\mathbb{S} = \{\mathbf{z} \in \mathcal{Z}, \mid f(\mathbf{z}) \in \mathcal{Y}\} \quad \rightarrow \underline{\mathbb{S}} \subseteq \mathbb{S} \subseteq \overline{\mathbb{S}}$$

$f([\mathbf{z}]) \subseteq \mathcal{Y}$ $\Rightarrow [\mathbf{z}] \subseteq \underline{\mathbb{S}}$: inner approximation

$f([\mathbf{z}]) \cap \mathcal{Y} = \emptyset$ $\Rightarrow [\mathbf{z}] \not\subseteq \overline{\mathbb{S}}$: outer approximation $\Rightarrow [\mathbf{z}] \subseteq \mathcal{Z} \backslash \overline{\mathbb{S}}$

otherwise partition . . .



UNIVERSITE D'ORLEANS

Set Membership Estimation

- State estimation with Continuous systems
  - Interval observers
  - Prediction-correction / Filtering approaches
    - Reachability + Set inversion

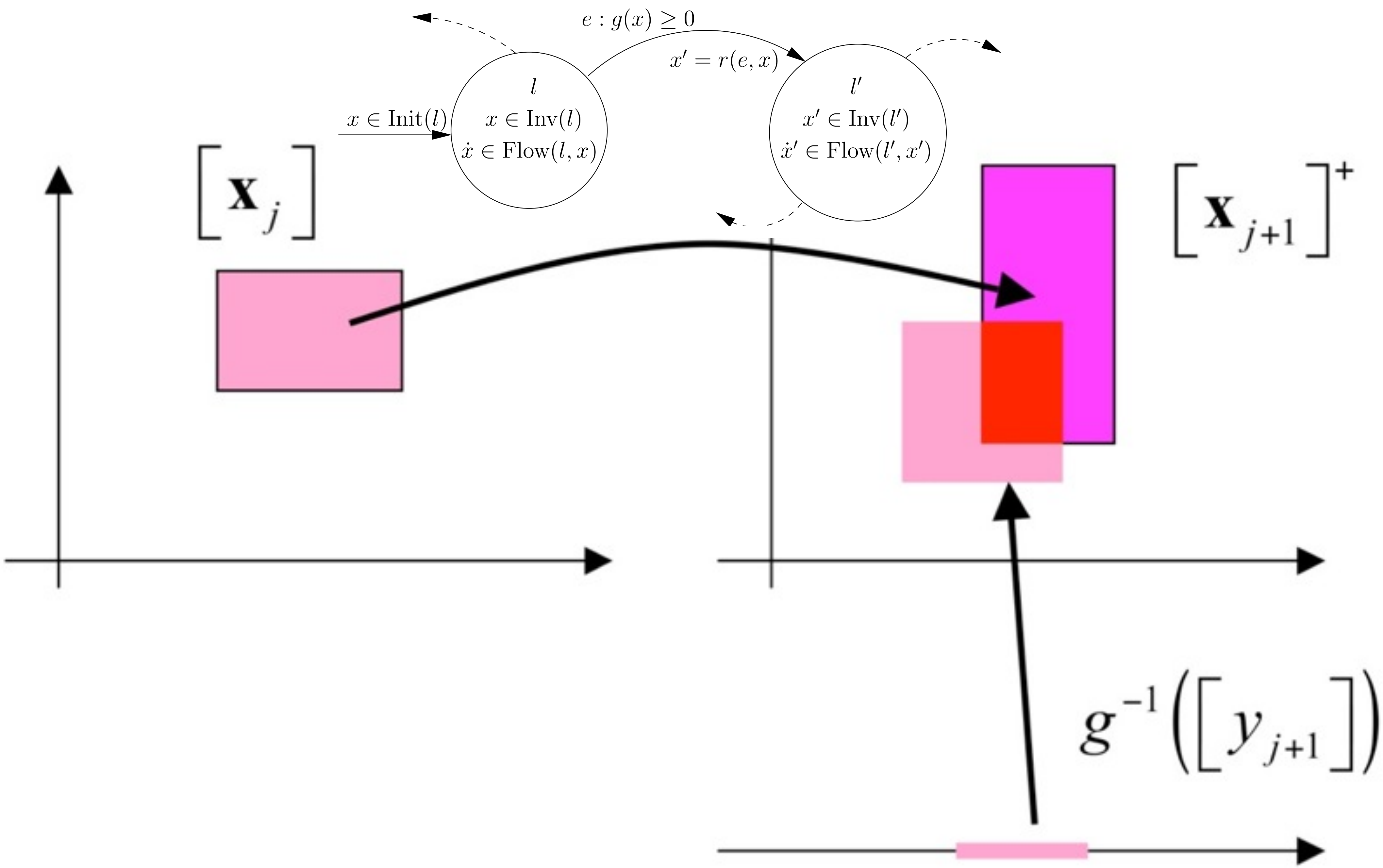
- **State estimation with Hybrid systems**
  - Piecewise affine systems (Bemporad, et al. 2005)
  - ODE + CSP (Goldsztejn, et al., 2010)
  - Nonlinear case (Benazera & Travé-Massuyès, 2009)
  - SAT mod ODE (Eggers, et al., 2012) (Maïga, et al. 2015).

Outline

- Hybrid dynamical systems
- Set membership estimation
- **Hybrid reachability approach**
- Example
- Research directions

Reachability based approach

- **Predictor-Corrector** approach for hybrid systems



Reachability based approach

- **Predictor-Corrector** approach for hybrid systems

MAGIIC SPS

LAAS-CNRS

UNIVERSITE D'ORLEANS

Hybrid Reachability Computation

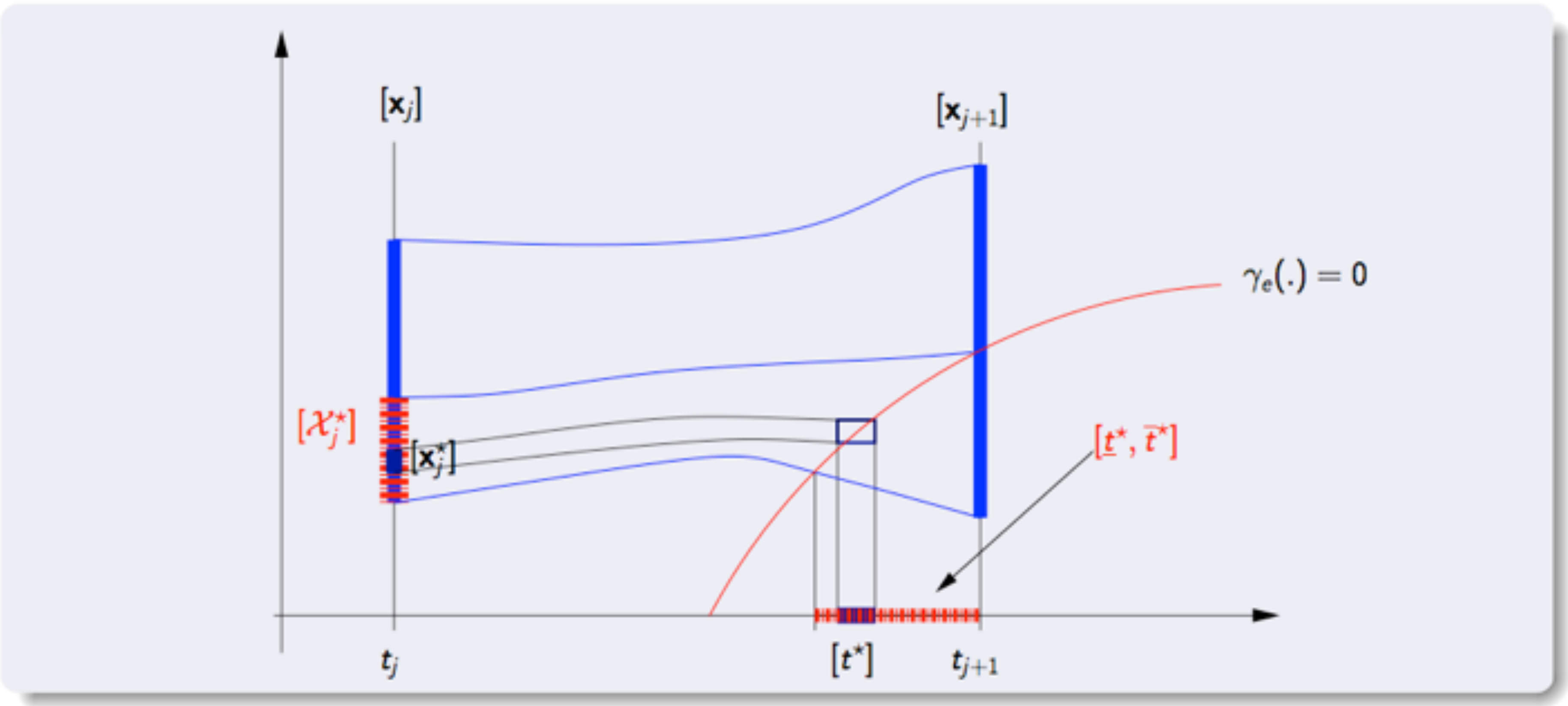
- **Guaranteed event detection & localization**
  - **An interval constraint propagation approach**
    - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

Hybrid Reachability Computation

Guaranteed event detection & localization

- **An interval constraint propagation approach**
  - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

Time grid $\rightarrow$ $t_0 < t_1 < t_2 < \cdots < t_N$



Compute $[\underline{t}^\star, \overline{t}^\star] \times [\mathcal{X}_j^\star]$

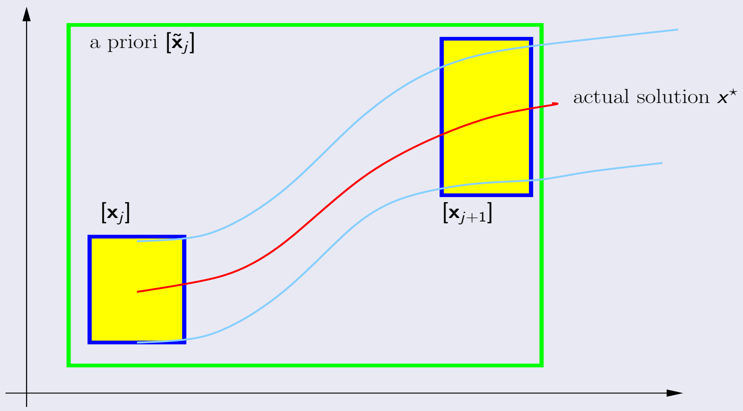
Hybrid Reachability Computation

Guaranteed event detection & localization

- **An interval constraint propagation approach**
  - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}, \mathbf{p}, t), \quad t_0 \leq t \leq t_N, \ \mathbf{x}(t_0) \in [\mathbf{x}_0], \ \mathbf{p} \in [\mathbf{p}]$$

Time grid $\longrightarrow$ $\quad t_0 < t_1 < t_2 < \cdots < t_N$



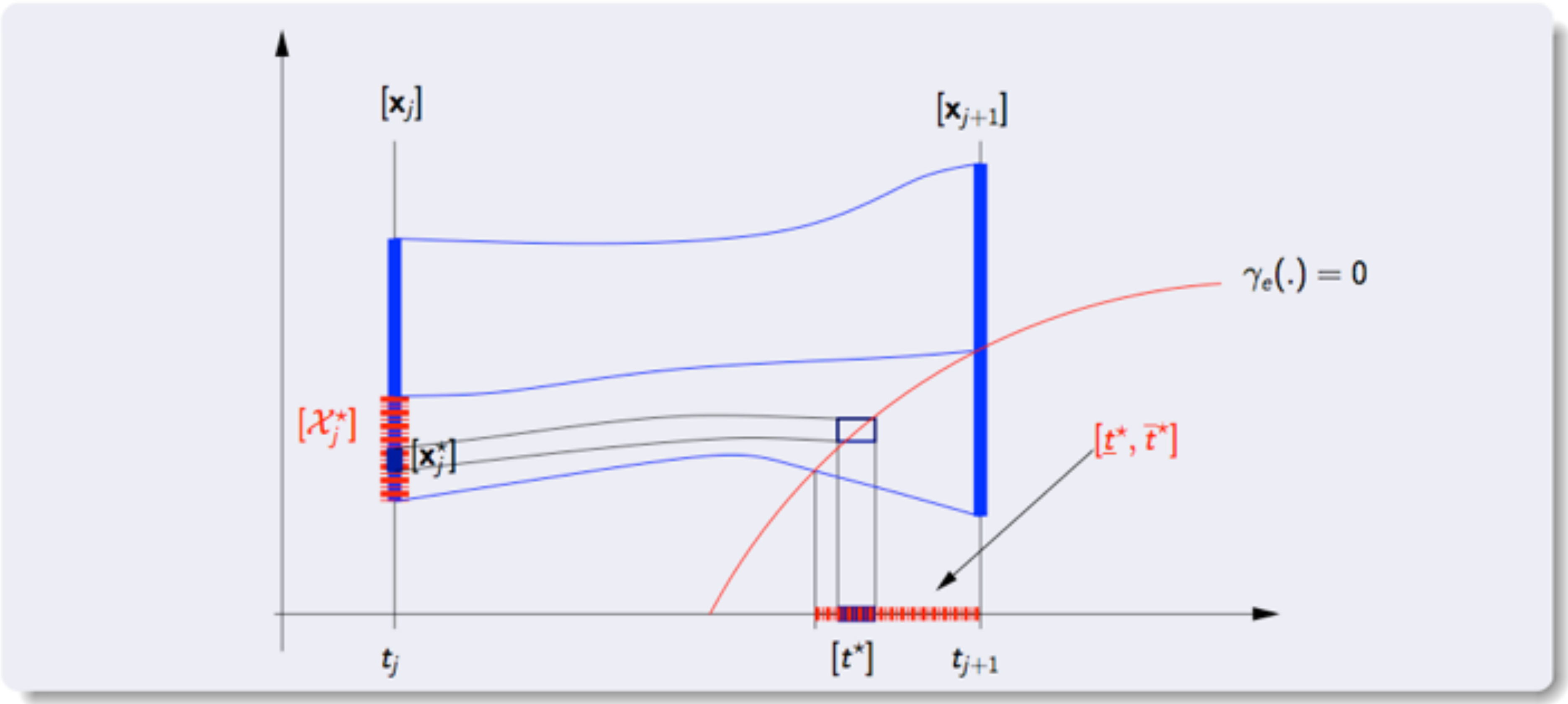
- Analytical solution for $[\mathbf{x}](t), \ t \in [t_j, t_{j+1}]$

$$[\mathbf{x}](t) = [\mathbf{x}_j] + \sum_{i=1}^{k-1} (t - t_j)^i \mathbf{f}^{[i]}([\mathbf{x}_j], [\mathbf{p}]) + (t - t_j)^k \mathbf{f}^{[k]}([\tilde{\mathbf{x}}_j], [\mathbf{p}])$$

Hybrid Reachability Computation

- **Guaranteed event detection & localization**
  - **An interval constraint propagation approach**
    - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

Time grid $\rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N$



Compute $[\underline{t}^\star, \overline{t}^\star] \times [\mathcal{X}_j^\star]$

Hybrid Reachability Computation

Guaranteed event detection & localization

- **An interval constraint propagation approach**
  - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

Time grid $\rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N$

- $[\mathbf{x}](t) = \text{Interval Taylor Series (ITS)}(t, [\mathbf{x}_j], [\tilde{\mathbf{x}}_j])$
- $\gamma([\mathbf{x}](t)) = 0$

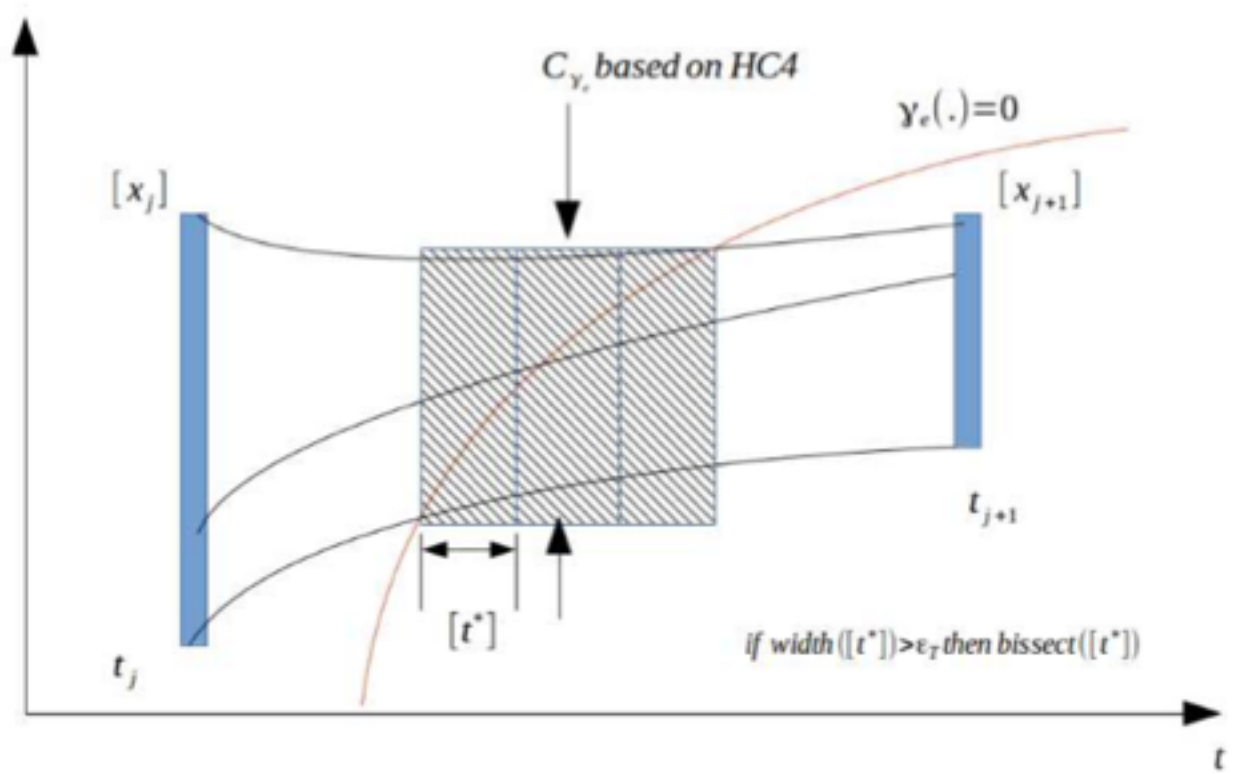
$\Rightarrow \gamma \circ \text{ITS}(t, \mathbf{x}_j, [\tilde{\mathbf{x}}_j]) \rightarrow \psi(t, \mathbf{x}_j)$

$$\text{Solve CSP } ([t_j, t_{j+1}] \times [\mathbf{x}_j], \psi(.,.) \ni 0)$$

Hybrid Reachability Computation

Detecting and localizing events

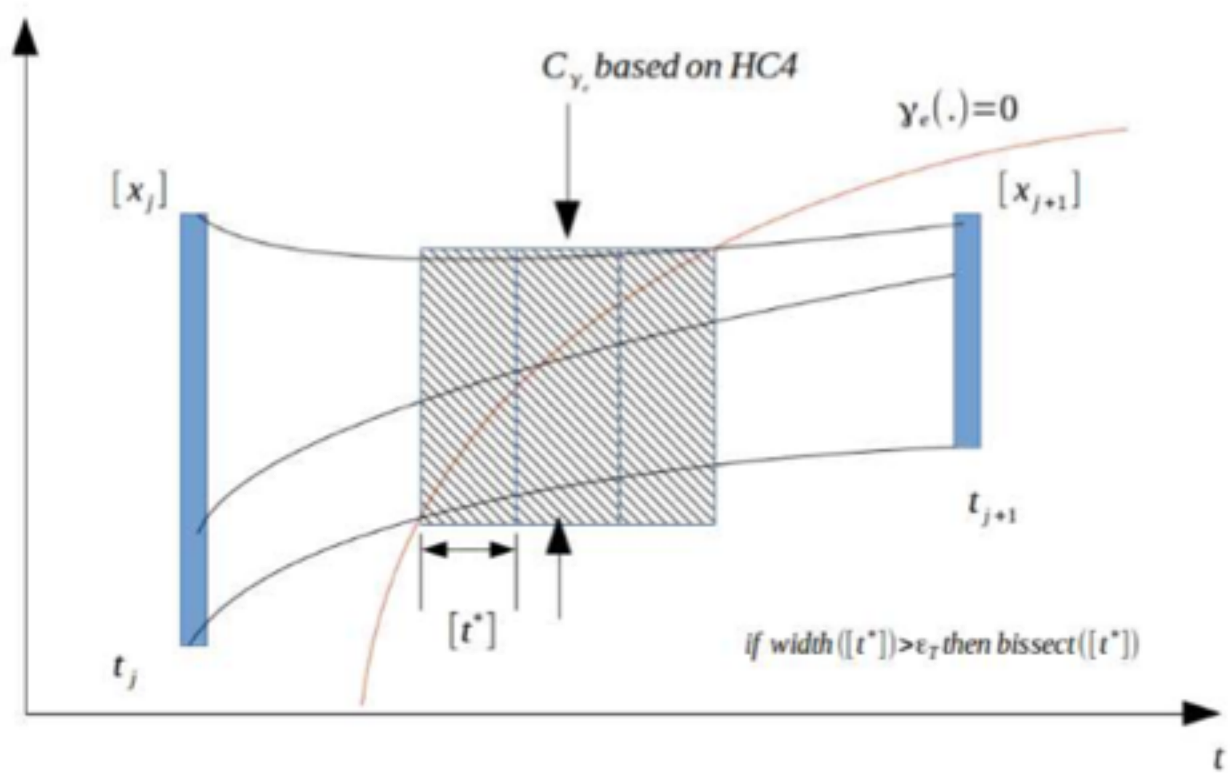
- **Improved and enhanced version. A faster version.**
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

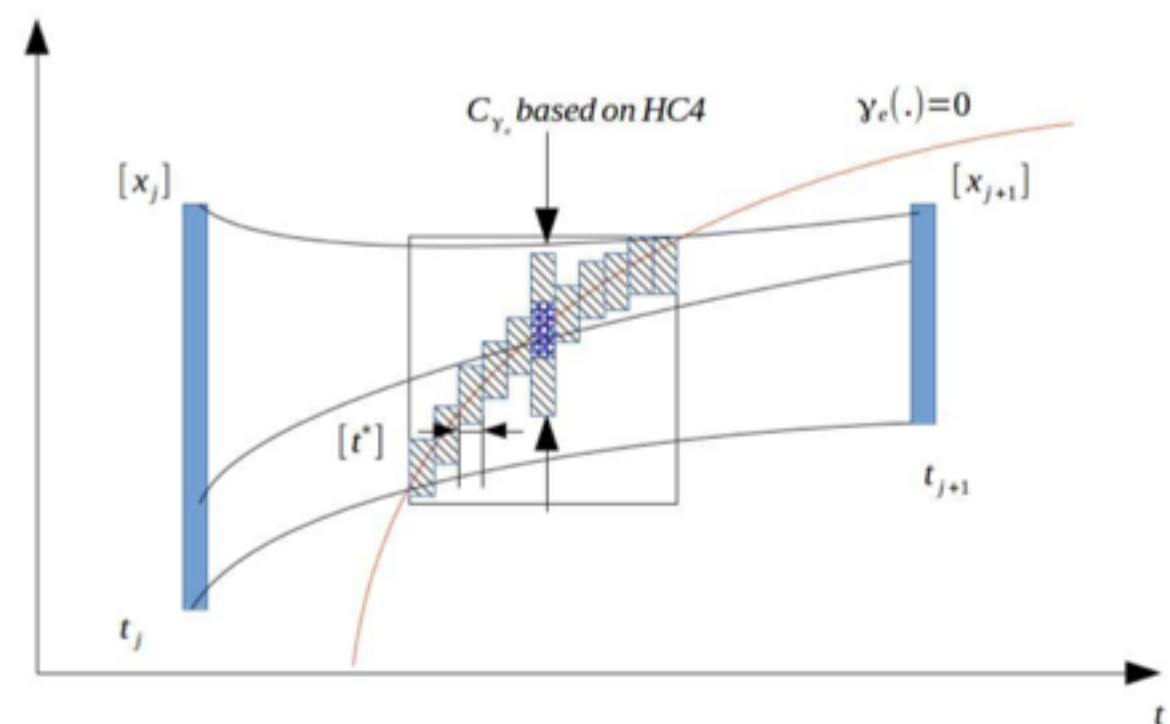


Hybrid Reachability Computation

Detecting and localizing events

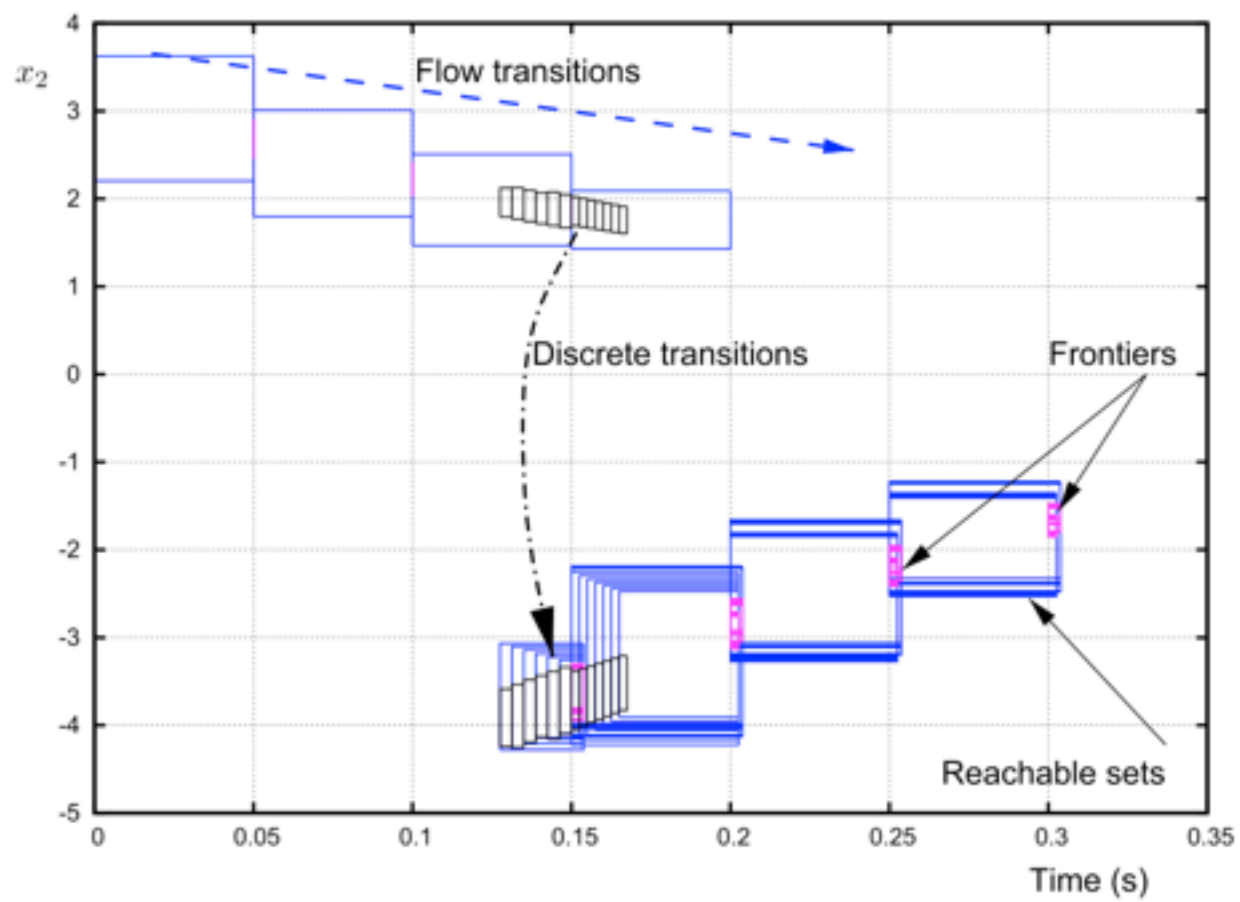
- **Improved and enhanced version. A faster version.**
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

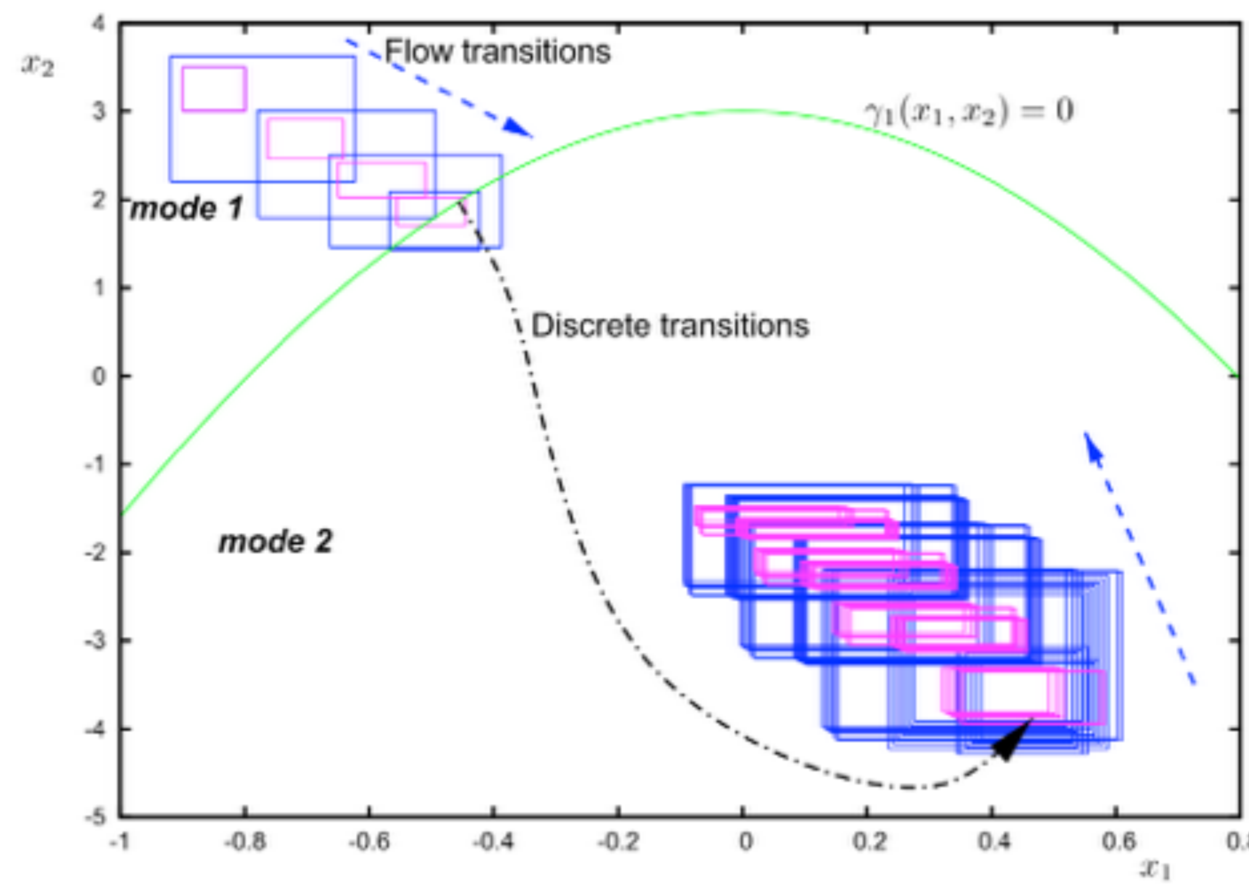




Hybrid Reachability Computation

- **Detecting and localizing events**
  - **Improved and enhanced version**
    - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

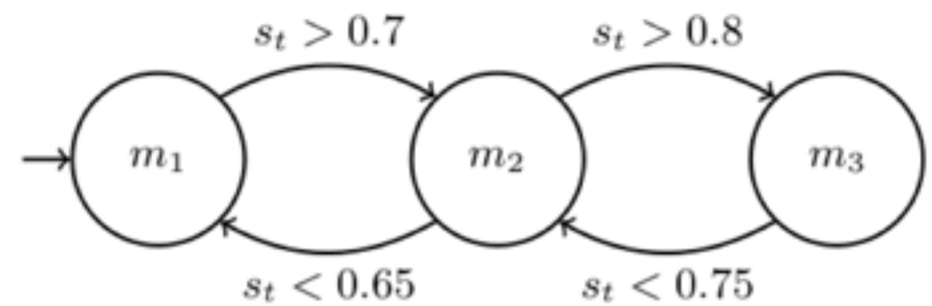


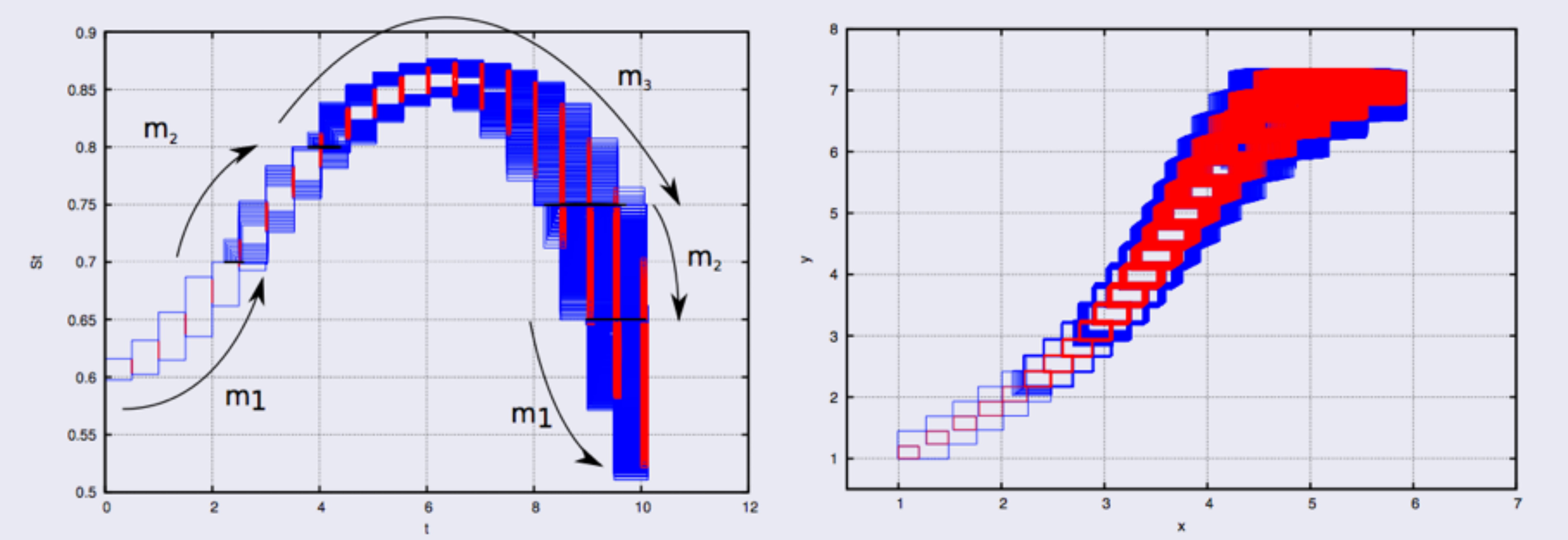


Hybrid Reachability Computation

Detecting and localizing events

- **Improved and enhanced version**
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)





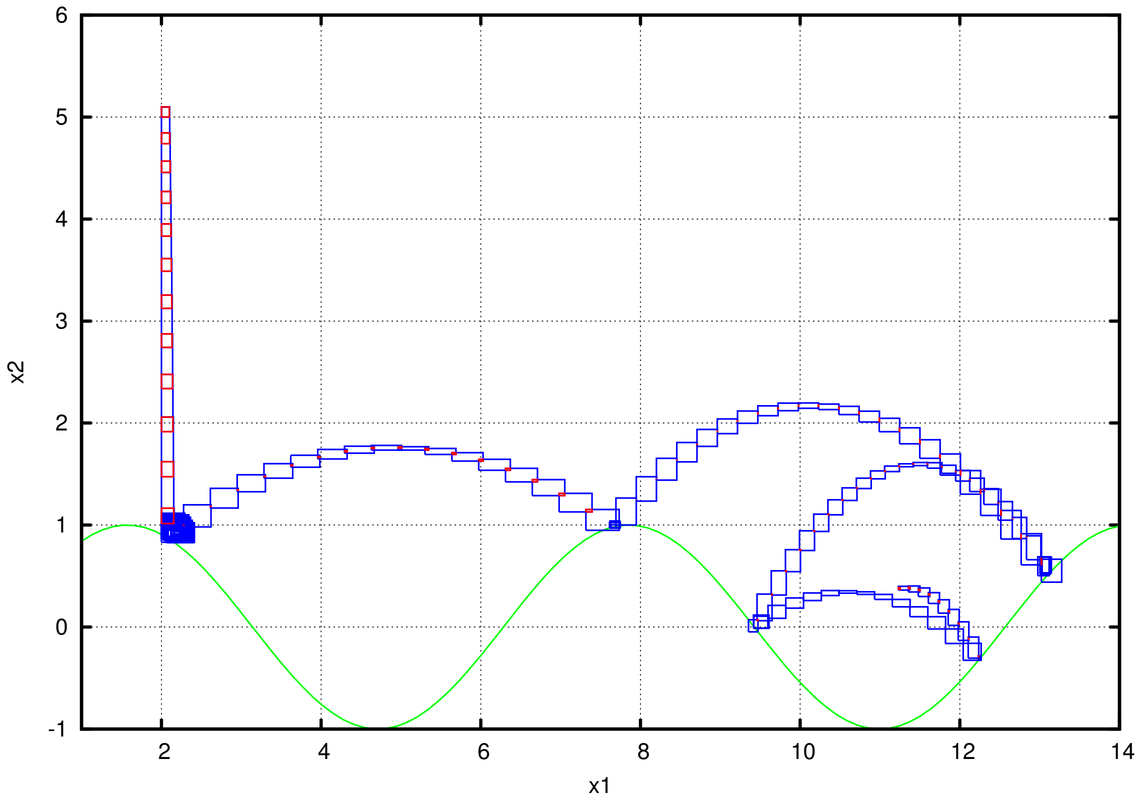
(e) St×t (f) Y×X space

Hybrid Reachability Computation

Detecting and localizing events

- **Improved and enhanced version**
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

Bouncing ball in 2D.

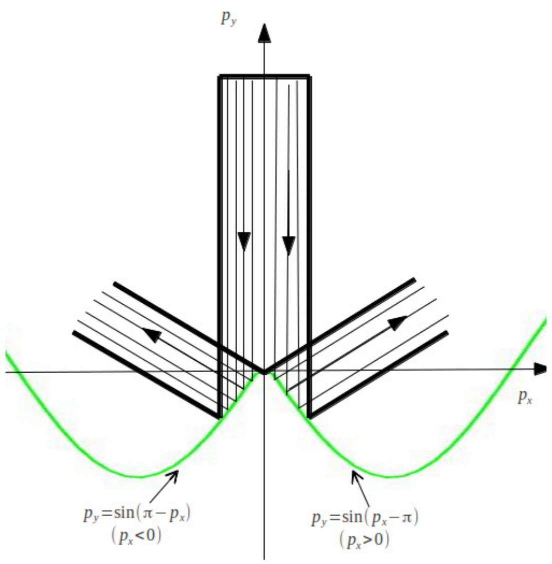


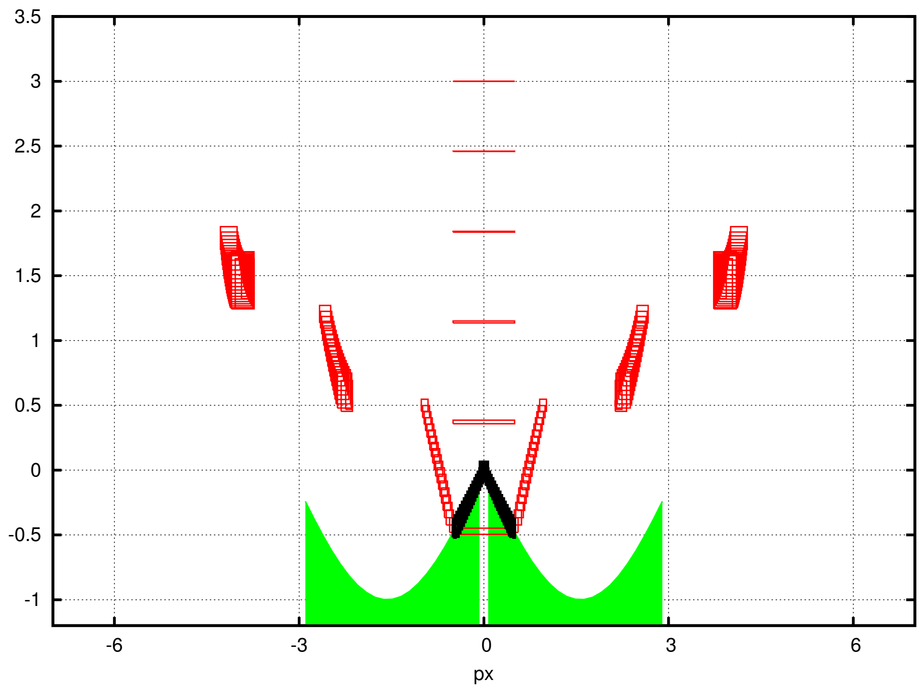
Hybrid Reachability Computation

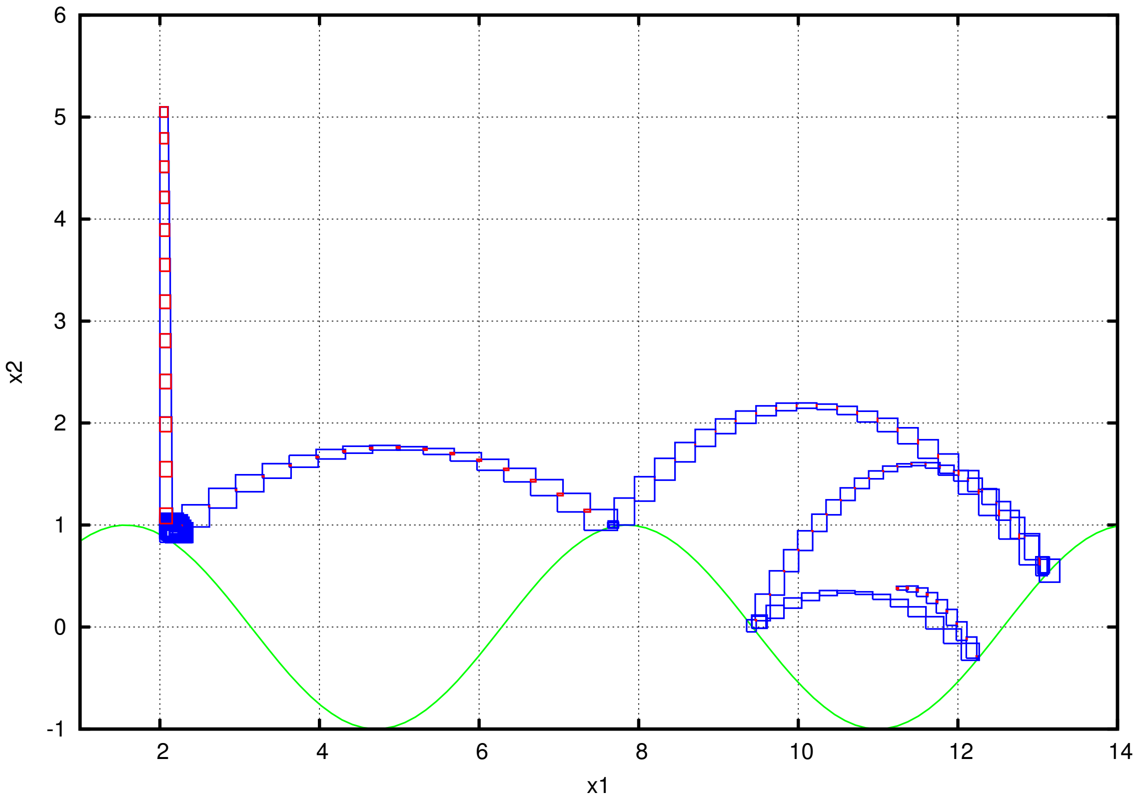
Detecting and localizing events

- **Improved and enhanced version**
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

Bouncing ball in 2D.







Set Membership Estimation

- **Parameter estimation with hybrid systems**
  - Branch-&-bound, branch-&-prune, interval contractors …
    (Eggers, Ramdani et al., 2012), (Maïga, Ramdani et al., 2015)

$$\begin{aligned}\mathbb{S} = \{\mathbf{p} \in \mathbb{P}_0 | \quad & (\forall t \in [t_0, T_{end}], \\ & flow(q) \wedge Inv(q) \wedge guard(e)) \\ & \wedge \forall t_j \in \{t_1, t_2, ..., T_n\}, g_q(x, \mathbf{p}, t) \in \mathbb{Y}_j \quad \}\end{aligned}$$

Set Membership Estimation

- **Parameter estimation with hybrid systems**
  - Branch-&-bound, branch-&-prune, interval contractors …
    (Eggers, Ramdani et al., 2012), (Maïga, Ramdani et al., 2015)

$$\begin{aligned}\mathbb{S} = \{\mathbf{p} \in \mathbb{P}_0 | \quad & (\forall t \in [t_0, T_{end}], \\ & flow(q) \wedge Inv(q) \wedge guard(e)) \\ & \wedge \forall t_j \in \{t_1, t_2, ..., T_n\}, g_q(x, \mathbf{p}, t) \in \mathbb{Y}_j \quad \}\end{aligned}$$

$$\underline{\mathbb{S}} \subseteq \mathbb{S} \subseteq \underline{\mathbb{S}} \cup \Delta\mathbb{S} \equiv \overline{\mathbb{S}}$$

Need an inclusion test!

Inclusion test

Zonotope $\quad \mathsf{Z} = c \oplus R\boldsymbol{B}^p$

Strip $\quad \mathcal{S}_j = \{x \in \mathbb{R}^n | \; |\eta^\top x - d_j| \leq \sigma_j\} \equiv [y_j]$

Zonotope support strip $\quad \mathcal{S}_{\mathsf{Z}} = \{x \in \mathbb{R}^n | \; q_d \leq \eta^\top x \leq q_u\}$

$$q_u = \min_{x \in \mathsf{Z}} \eta^\top x = \eta^\top c - \|R^\top \eta\|_1$$

$$q_d = \max_{x \in \mathsf{Z}} \eta^\top x = \eta^\top c + \|R^\top \eta\|_1$$

Theorem [(Vicino and Zappa (1996))]

$$\mathsf{Z} \cap \mathcal{S}_j = \emptyset \iff (q_d \geq d_j - \sigma_j) \wedge (q_u \leq d_j + \sigma_j)$$

$$\mathsf{Z} \subseteq \mathcal{S}_j \iff (q_u < d_j - \sigma_j) \vee (q_d > d_j + \sigma_j)$$

Inclusion test

UNIVERSITE D'ORLEANS

$$\text{Zonotope} \quad \mathsf{Z} = c \oplus R\boldsymbol{B}^p$$

$$\text{Strip} \quad \mathcal{S}_j = \{x \in \mathbb{R}^n |\ |\eta^\top x - d_j| \leq \sigma_j\} \equiv [y_j]$$

$$\boxed{\text{Zonotope support strip} \quad \mathcal{S}_{\mathsf{Z}} = \{x \in \mathbb{R}^n |\ q_d \leq \eta^\top x \leq q_u\}}$$

$$q_u = \min_{x \in \mathsf{Z}} \eta^\top x = \eta^\top c - \|R^\top \eta\|_1$$

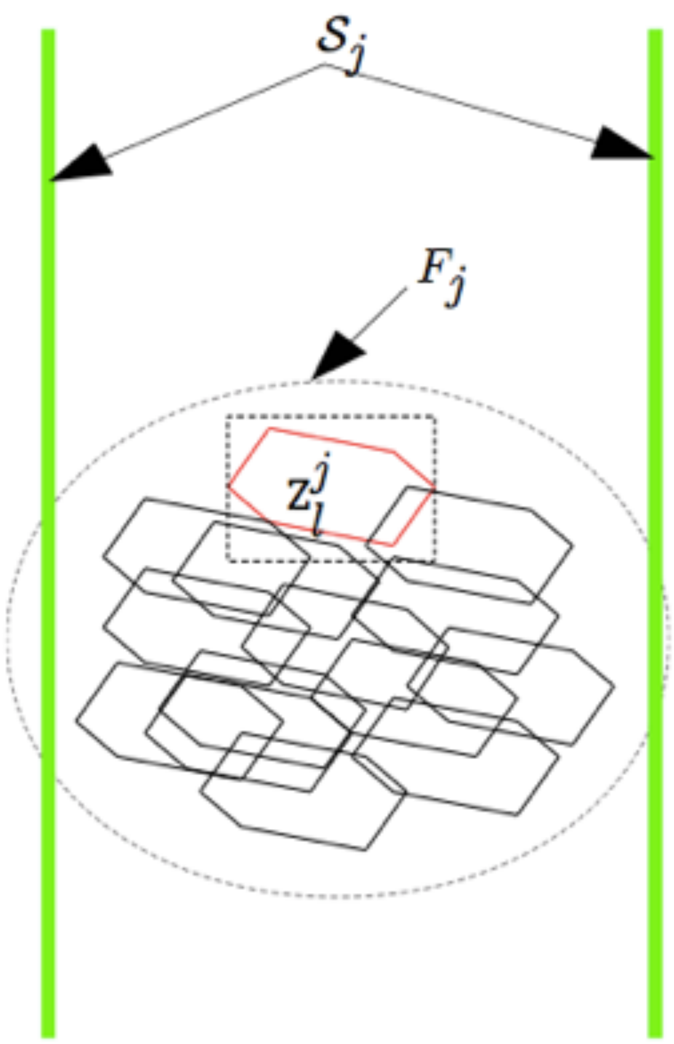
$$q_d = \max_{x \in \mathsf{Z}} \eta^\top x = \eta^\top c + \|R^\top \eta\|_1$$

Theorem [(Vicino and Zappa (1996))]

$$\boxed{\begin{aligned} \mathsf{Z} \cap \mathcal{S}_j = \emptyset &\iff (q_d \geq d_j - \sigma_j) \wedge (q_u \leq d_j + \sigma_j) \\ \mathsf{Z} \subseteq \mathcal{S}_j &\iff (q_u < d_j - \sigma_j) \vee (q_d > d_j + \sigma_j) \end{aligned}}$$

Inclusion test

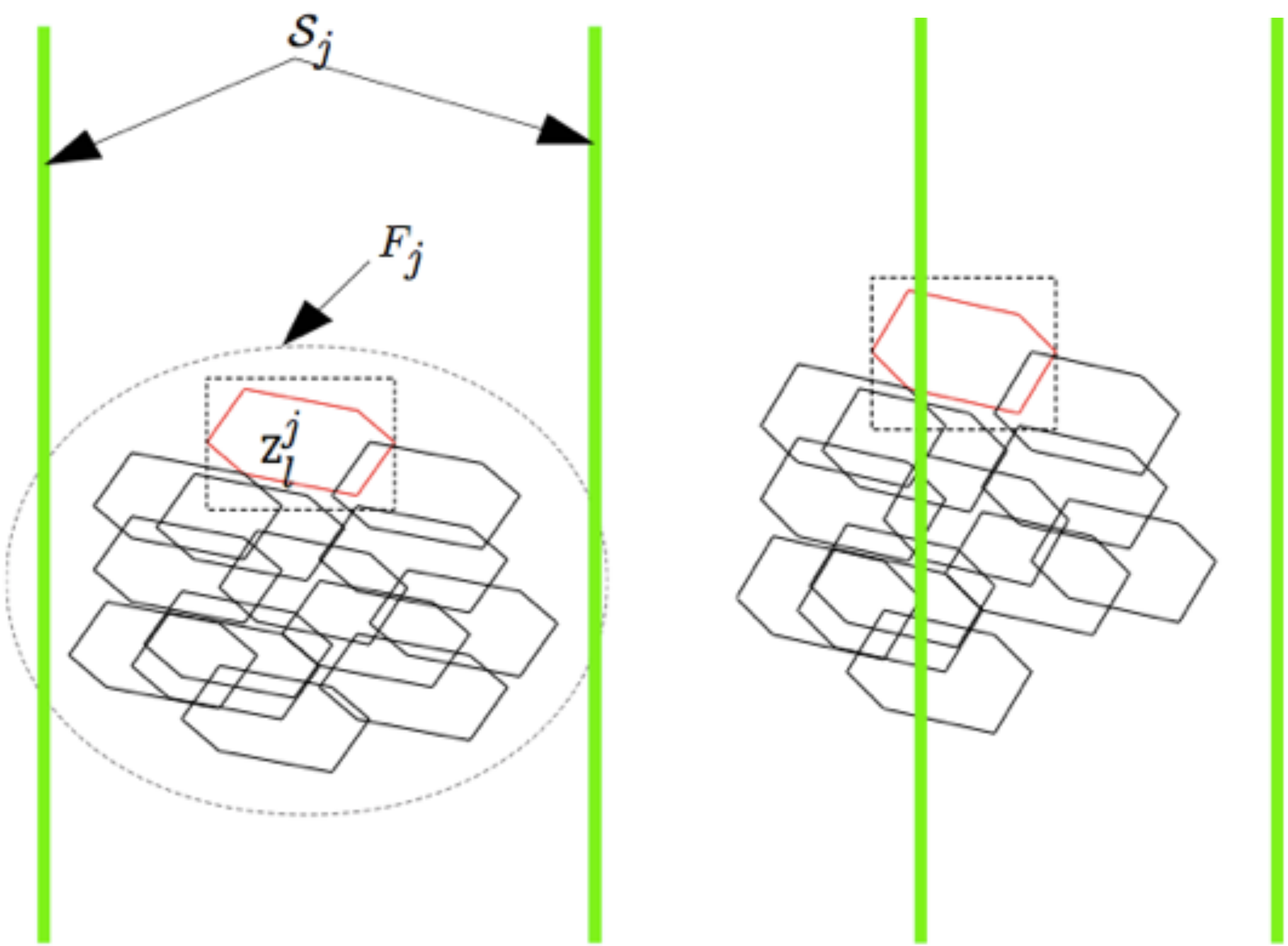
Frontier of the reachable set = union of zonotopes



(a) Test: is true

Inclusion test

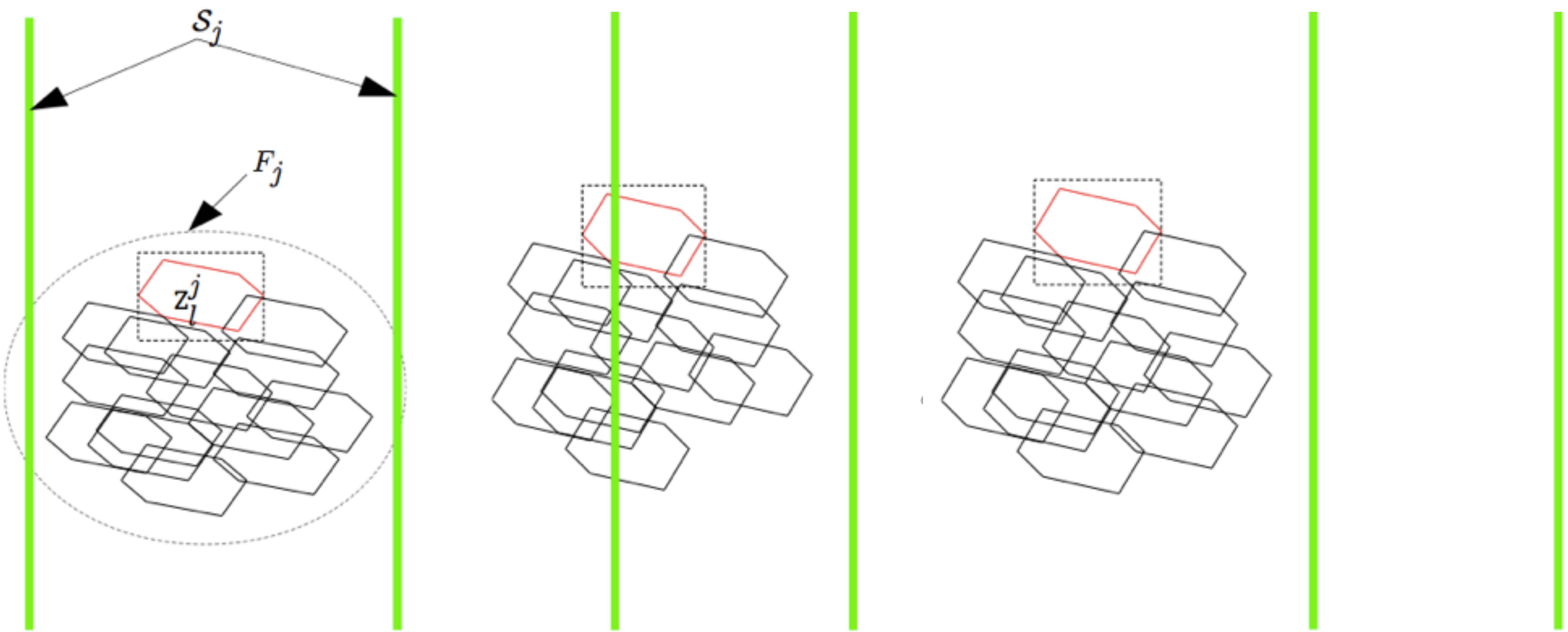
Frontier of the reachable set = union of zonotopes



(a) Test: is true   (b) is ambiguous

Inclusion test

Frontier of the reachable set = union of zonotopes



(a) Test: is true (b) is ambiguous (c) is false

Outline

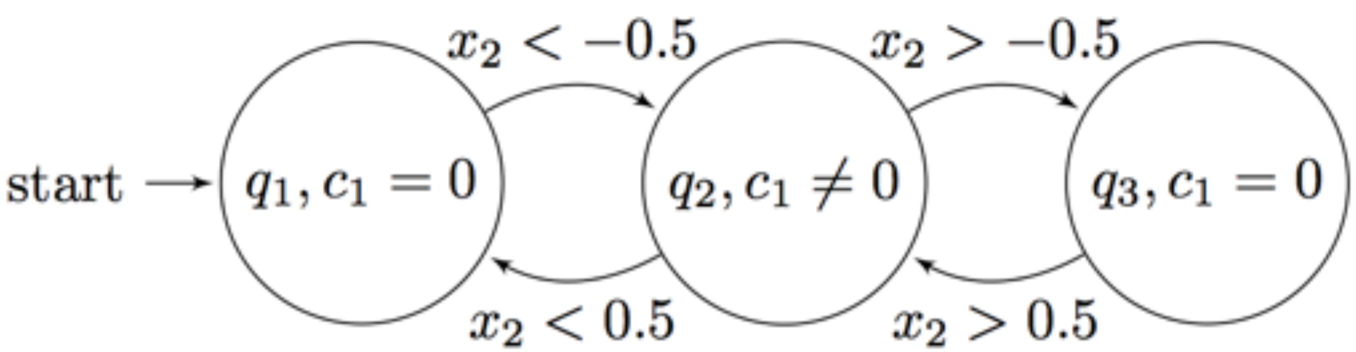
- Hybrid dynamical systems
- Set membership estimation
- Hybrid reachability approach
- **Example**
- Research directions

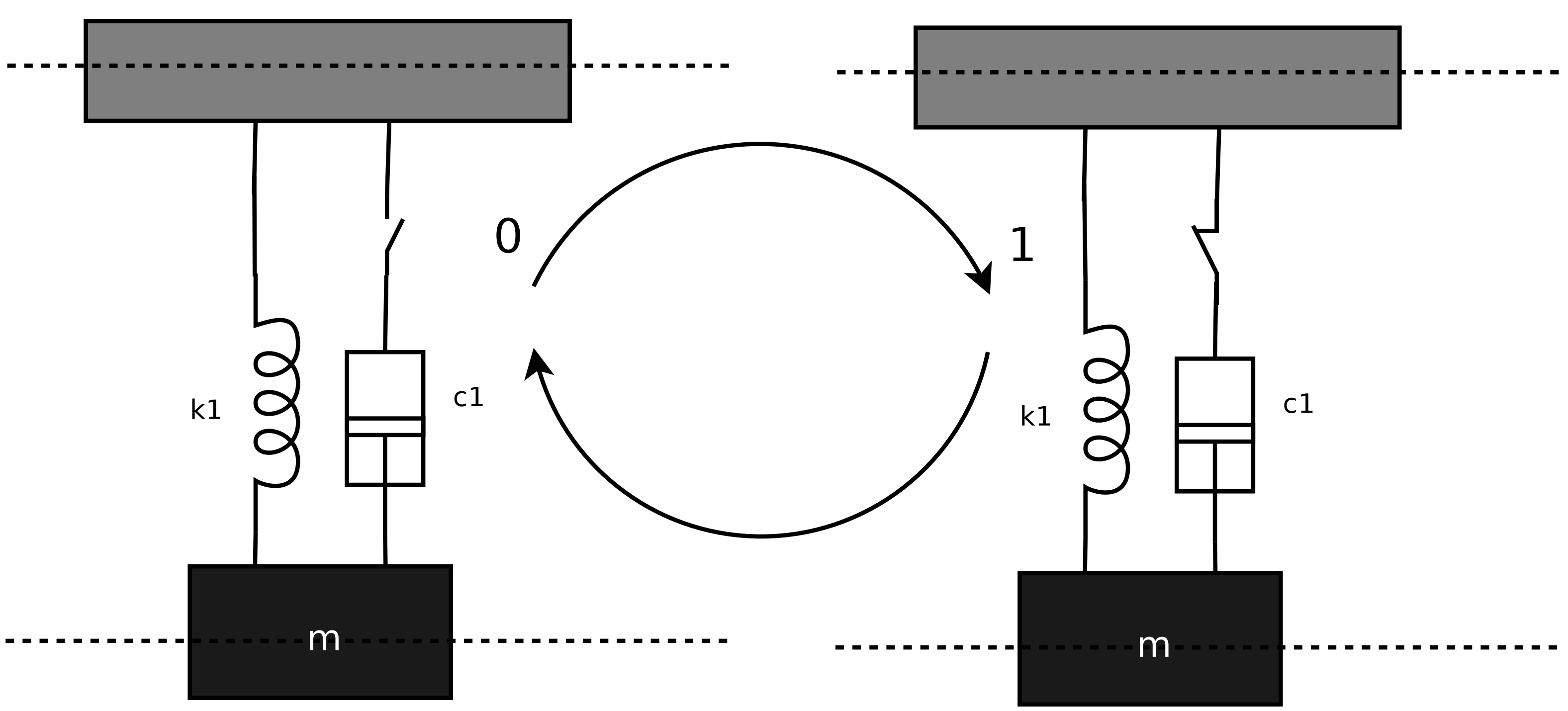
UNIVERSITE D'ORLEANS

Parameter identification

Hybrid Mass-Spring

- Velocity-dependent damping. Mode switching driven by velocity.

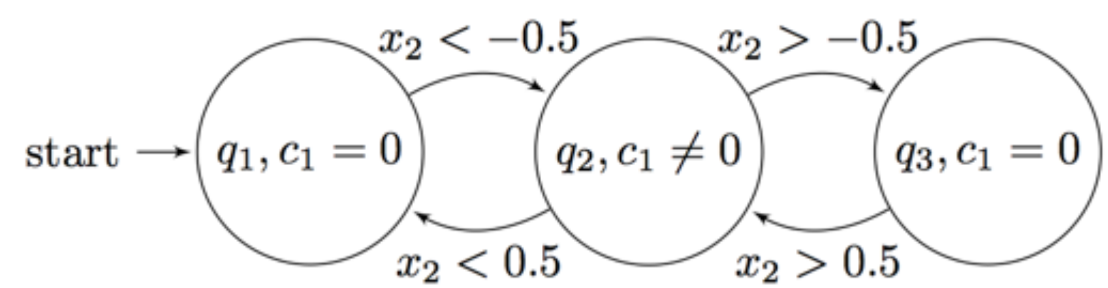


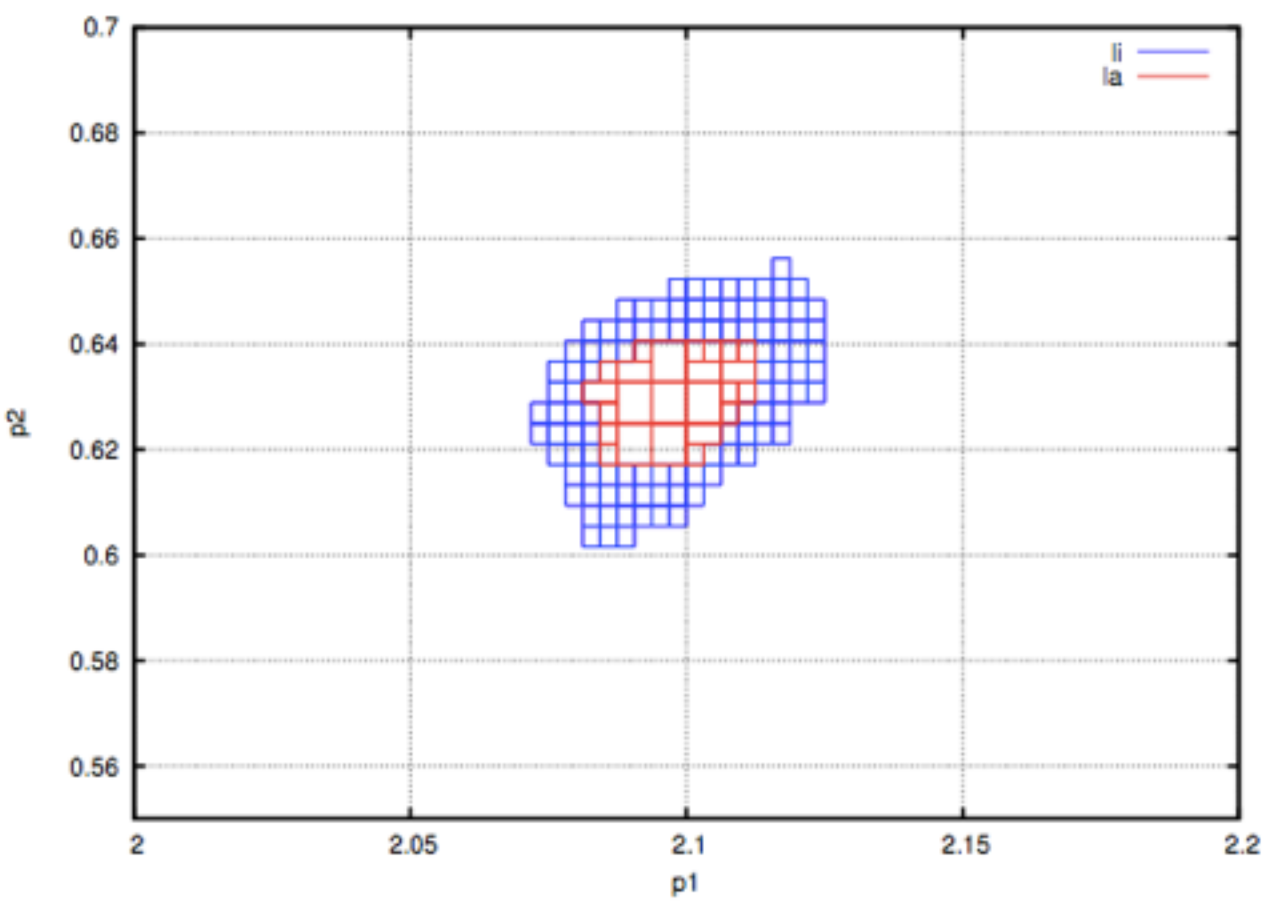


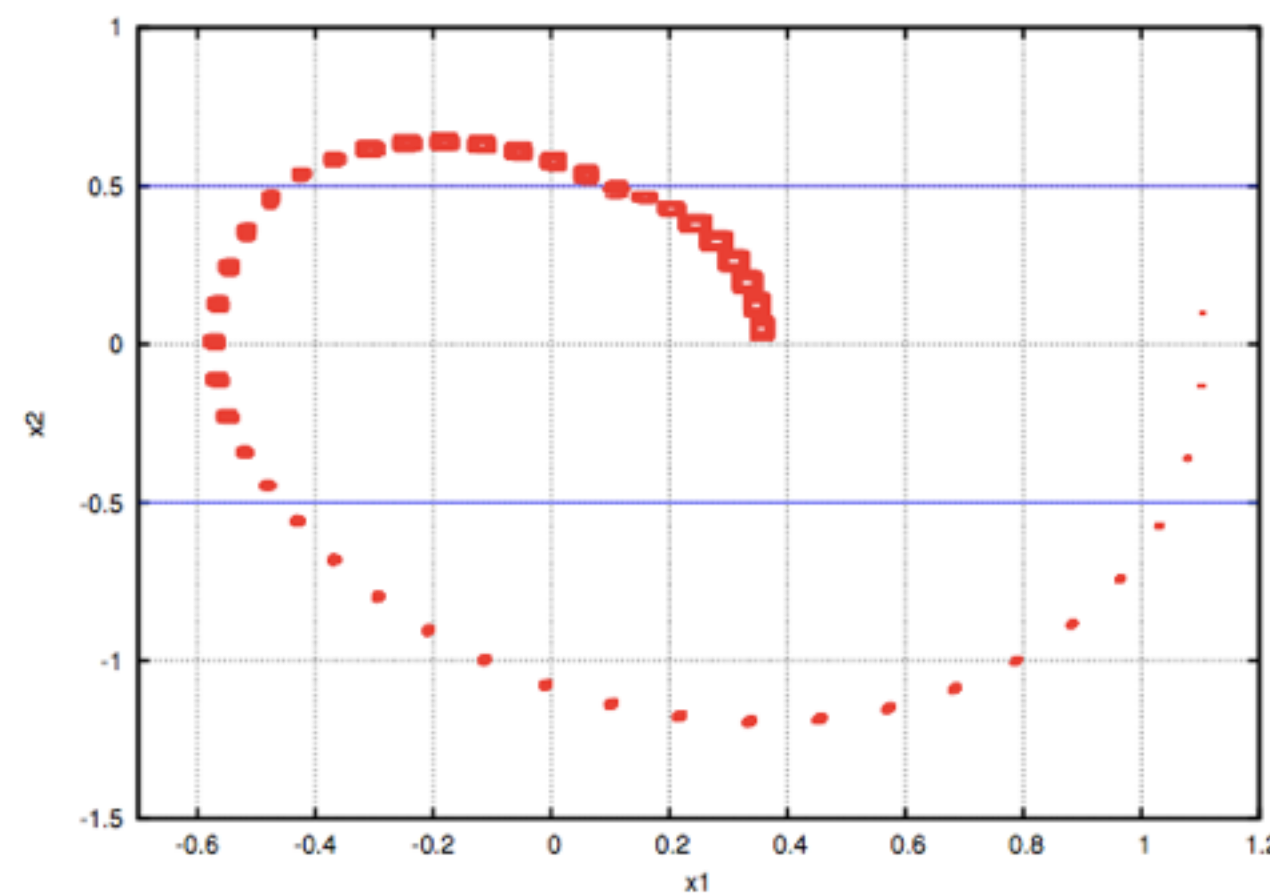
Parameter identification

Hybrid Mass-Spring

- **case 1 : Parameters acting on continuous dynamics.**
  - ***CPU time approx. 140 mn!***



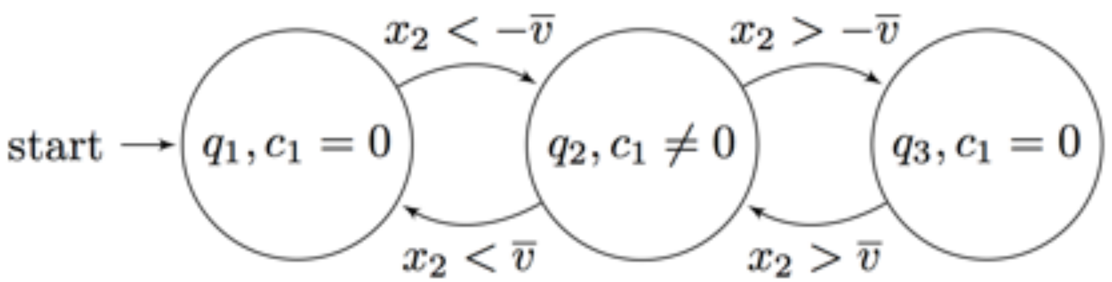


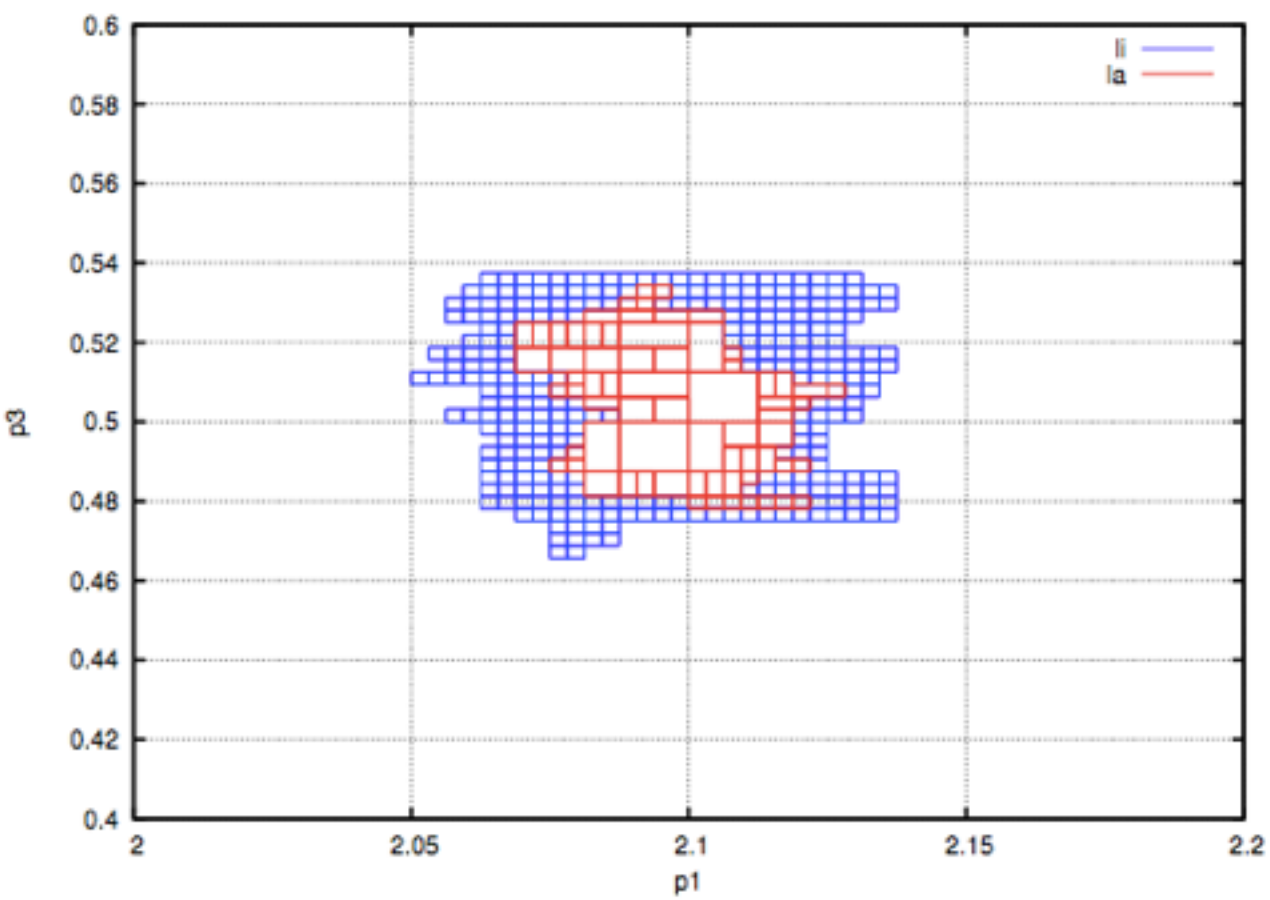


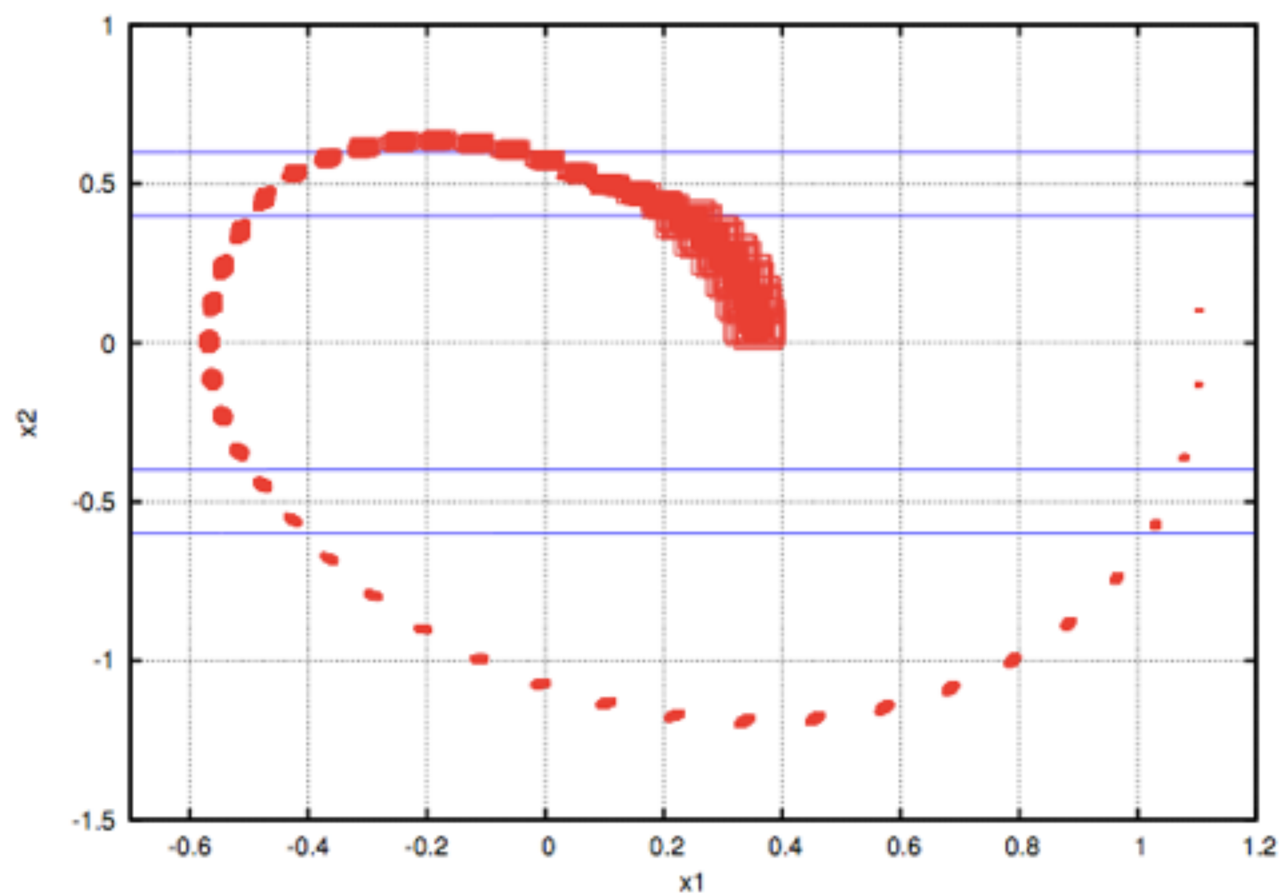
Parameter identification

Hybrid Mass-Spring

- **case 2 : parameters acting on discrete transition.**
  - ***CPU time approx. 40 mn***







Outline

- Hybrid dynamical systems
- Set membership estimation
- Hybrid reachability approach
- Example
- **Research directions**

Research directions

- **Contractors** for hybrid dynamical systems
  - To build upon a hybrid reachability approach
- Effective methods for set membership **estimation**
  - SM parameter estimation …
  - SM **hybrid** state estimation of nonlinear hybrid systems
- Combine with decision making for FDI
  - Application to actual hybrid systems

Focused References

- N. Ramdani and N. S.Nedialkov, Computing reachable sets for uncertain nonlinear hybrid systems using interval constraint propagation techniques, **Nonlinear Analysis: Hybrid Systems**, 5(2), pp.149-162, **2011**.
- A.Eggers, N.Ramdani, N.S.Nedialkov, M.Fränzle, Set-Membership Estimation of Hybrid Systems via SAT Mod ODE. in **IFAC SYSID 2012**. pp.440-445
- M. Maïga, N. Ramdani, L. Travé-Massuyès, A fast method for solving guard set intersection in nonlinear hybrid reachability, in **IEEE CDC 2013**, pp.508-513.
- M. Maïga, C. Combastel, N. Ramdani, L. Travé-Massuyès, Nonlinear hybrid reachability using set integration and zonotope enclosures. in **ECC 2014**, pp.234-239.
- M. Maïga, N. Ramdani, L. Travé-Massuyès, C. Combastel, A CSP versus a zonotope-based method for solving guard set intersection in nonlinear hybrid reachability, **Mathematics in Computer Science**, **2014.**
- M. Maïga, N. Ramdani, L. Travé-Massuyès, Robust fault detection in hybrid systems using set-membership parameter estimation, in **IFAC SafeProcess 2015**, Paris.