Parameter identification with hybrid systems in a bounded-error framework

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SWIM 2015, Praha
9-11 June 2015
Model-based FDI

- Extend to hybrid dynamical systems set-membership approaches for model-based FDI
Outline

- Hybrid dynamical systems
- Set membership estimation
- Hybrid reachability approach
- Example
- Research directions
Hybrid Cyber-Physical Systems

- Interaction discrete + continuous dynamics
- Safety-critical embedded systems
- Networked autonomous systems
Hybrid Cyber-Physical Systems

Modelling $\rightarrow$ hybrid automaton (Alur, et al. 1995)

- Non-linear continuous dynamics
- Bounded uncertainty

$H = (Q, \mathcal{D}, \mathcal{P}, \Sigma, \mathcal{A}, \text{Inv}, \mathcal{F})$,

**Continuous dynamics**

$\dot{x}(t) = f_q(x, p, t)$,
$\nu_q(x(t), p, t) < 0$,

$e : g(x) \geq 0$

$x' = r(e, x)$

$\dot{x}' \in \text{Flow}(l', x')$

$x' \in \text{Inv}(l')$

$H = (Q, \mathcal{D}, \mathcal{P}, \Sigma, \mathcal{A}, \text{Inv}, \mathcal{F})$,

**Discrete dynamics**

$\mathcal{A} \ni e : (q \rightarrow q') = (q, \text{guard}, \sigma, \rho, q')$
$\gamma_e(x(t), p, t) = 0$

$t_0 \leq t \leq t_N$
$x(t_0) \in X_0 \subseteq \mathbb{R}^n$
$p \in \mathcal{P}$
Example: the bouncing ball

Initial conditions:
- $x = 10, v = 0$

Discrete transition:
- $x = 0$
- $v' := -v$

Graphs:
- Height $x$ vs. time
- Velocity $v$ vs. time
Example: the bouncing ball
Example: the bouncing ball
Modelling → hybrid automaton

- Nonlinear continuous dynamics
- Nonlinear guards sets
- Nonlinear reset functions

Bounded uncertainty
**Estimation of Hybrid State**

- **Modelling → hybrid automaton**
  - Nonlinear continuous dynamics
  - Nonlinear guards sets
  - Nonlinear reset functions
  - **Bounded** uncertainty

- **Faults as discrete modes !!**
Modelling → hybrid automaton
  - Nonlinear …
  - Bounded uncertainty

Faults as a mode !!

FDI → State Estimation
  → reconstruct system variables
    - switching sequence
    - continuous variables

Estimation of Hybrid State
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Classical estimation is probabilistic

Yield valid results only if
- Perturbations, errors and model uncertainties with statistical properties known \textit{a priori}
- Model structure is correct, no modeling errors
Unknown but bounded-error framework

Hypothesis
Uncertainties and errors are bounded with known prior bounds

A set of feasible solutions
\[ S = \{ p \in \mathbb{P} | f(p) \in Y \} = f^{-1}(Y) \cap \mathbb{P} \]
State estimation with continuous systems

- Interval observers
  - (Moisan, et al. 2009), (Meslem & Ramdani, 2011),
  - (Raïssi, et al., 2012), (El Thabet, et al. 2014) ....
State estimation with continuous systems

- Prediction - Correction / Filtering approaches
  - (Raïssi et al., 2005), (Meslem, et al, 2010),
    (Milanese & Novara, 2011), (Kieffer & Walter, 2011) …
Set Membership Estimation

- Set inversion. Parameter estimation
  - Branch-&-bound, branch-&-prune, interval contractors …
    (Jaulin, et al. 93) (Raïssi et al., 2004)

$$S = \{ z \in \mathbb{Z}, \ | \ f(z) \in \mathbb{Y} \} \rightarrow S \subseteq \mathbb{S} \subseteq \bar{S}$$

$$f([z]) \subseteq \mathbb{Y} \quad \Rightarrow [z] \subseteq \mathbb{S} : \text{inner approximation}$$
$$f([z]) \cap \mathbb{Y} = \emptyset \quad \Rightarrow [z] \notin \bar{S} : \text{outer approximation} \quad \Rightarrow [z] \subseteq \mathbb{Z}\backslash\bar{S}$$

otherwise

partition …
State estimation with Continuous systems

- Interval observers
- Prediction-correction / Filtering approaches
  - Reachability + Set inversion

State estimation with Hybrid systems

- Piecewise affine systems (Bemporad, et al. 2005)
- ODE + CSP (Goldsztejn, et al., 2010)
- Nonlinear case (Benazera & Travé-Massuyès, 2009)
Hybrid dynamical systems
Set membership estimation
Hybrid reachability approach
Example
Research directions
Reachability based approach

**Predictor-Corrector** approach for hybrid systems
Reachability based approach

- **Predictor-Corrector** approach for hybrid systems
Guaranteed event detection & localization

- An interval constraint propagation approach
  -(Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)
Hybrid Reachability Computation

- Guaranteed event detection & localization
  - An interval constraint propagation approach
    - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

Time grid → $t_0 < t_1 < t_2 < \cdots < t_N$

Compute $[t^*, \overline{t}^*] \times [\mathcal{X}^*_j]$
Guaranteed event detection & localization

- An interval constraint propagation approach
  - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

\[ \dot{x}(t) = f(x, p, t), \quad t_0 \leq t \leq t_N, \quad x(t_0) \in [x_0], \quad p \in [p] \]

Time grid \( t_0 < t_1 < t_2 < \cdots < t_N \)

Analytical solution for \([x](t), \quad t \in [t_j, t_{j+1}]\)

\[ [x](t) = [x_j] + \sum_{i=1}^{k-1} (t - t_j)^i f^i([x_j], [p]) + (t - t_j)^k f^k([	ilde{x}^*_j], [p]) \]
Hybrid Reachability Computation

- Guaranteed event detection & localization
  - An interval constraint propagation approach
    - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

Time grid $\rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N$

Compute $[t^*, \bar{t}^*] \times [x^*_j]$
Guaranteed event detection & localization

- An interval constraint propagation approach
  - (Ramdani, et al., Nonlinear Analysis Hybrid Systems 2011)

Time grid → \( t_0 < t_1 < t_2 < \cdots < t_N \)

\[ [x](t) = \text{Interval Taylor Series (ITS)}(t, [x_j], [\dot{x}_j]) \]

\[ \gamma([x](t)) = 0 \]

\[ \Rightarrow \gamma \circ \text{ITS}(t, x_j, [\dot{x}_j]) \rightarrow \psi(t, x_j) \]

Solve CSP \( ([t_j, t_{j+1}] \times [x_j], \psi(\ldots) \ni 0) \)
Detecting and localizing events

- Improved and enhanced version. A faster version.
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)
Detecting and localizing events

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Detecting and localizing events

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Bouncing ball in 2D.
Detecting and localizing events

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  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

Bouncing ball in 2D.
Set Membership Estimation

Parameter estimation with hybrid systems
- Branch-&-bound, branch-&-prune, interval contractors ... (Eggers, Ramdani et al., 2012), (Maïga, Ramdani et al., 2015)

\[
S = \{ \mathbf{p} \in P_0 | \ (\forall t \in [t_0, T_{end}], \\
\quad \text{flow}(q) \land \text{Inv}(q) \land \text{guard}(e)) \\
\land \forall t_j \in \{t_1, t_2, ..., T_n\}, g_q(x, \mathbf{p}, t) \in \mathbb{Y}_j \} 
\]
Parameter estimation with hybrid systems

- Branch-&-bound, branch-&-prune, interval contractors …
  (Eggers, Ramdani et al., 2012), (Maïga, Ramdani et al., 2015)

\[
S = \{ p \in \mathbb{P}_0 | \forall t \in [t_0, T_{\text{end}}], \\
flow(q) \land \text{Inv}(q) \land \text{guard}(e) \\
\land \forall t_j \in \{t_1, t_2, ..., T_n\}, g_q(x, p, t) \in \mathbb{Y}_j \}
\]

\[S \subseteq S' \subseteq S \cup \Delta S \equiv \bar{S}\]

Need an inclusion test!
Zonotope $Z = c \oplus RB^p$
Strip $S_j = \{ x \in \mathbb{R}^n \mid \eta^\top x - d_j \mid \leq \sigma_j \} \equiv [y_j]$

Zonotope support strip $S_Z = \{ x \in \mathbb{R}^n \mid q_d \leq \eta^\top x \leq q_u \}$
$q_u = \min_{x \in Z} \eta^\top x = \eta^\top c - \| R^\top \eta \|_1$
$q_d = \max_{x \in Z} \eta^\top x = \eta^\top c + \| R^\top \eta \|_1$

Theorem [(Vicino and Zappa (1996))]$
Z \cap S_j = \emptyset \iff (q_d \geq d_j - \sigma_j) \land (q_u \leq d_j + \sigma_j)$
$Z \subseteq S_j \iff (q_u < d_j - \sigma_j) \lor (q_d > d_j + \sigma_j)$
Zonotope  $Z = c \oplus R B^p$

Strip  $S_j = \{ x \in \mathbb{R}^n | |\eta^\top x - d_j| \leq \sigma_j \} \equiv [y_j]$

Zonotope support strip  $S_z = \{ x \in \mathbb{R}^n | q_d \leq \eta^\top x \leq q_u \}$

\[ q_u = \min_{x \in Z} \eta^\top x = \eta^\top c - \| R^\top \eta \|_1 \]

\[ q_d = \max_{x \in Z} \eta^\top x = \eta^\top c + \| R^\top \eta \|_1 \]

Theorem [(Vicino and Zappa (1996))]

$Z \cap S_j = \emptyset \iff (q_d \geq d_j - \sigma_j) \land (q_u \leq d_j + \sigma_j)$

$Z \subseteq S_j \iff (q_u < d_j - \sigma_j) \lor (q_d > d_j + \sigma_j)$
Frontier of the reachable set = union of zonotopes

(a) Test: is true
Frontier of the reachable set = union of zonotopes

(a) Test: is true  (b) is ambiguous
Frontier of the reachable set = union of zonotopes
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Hybrid Mass-Spring

- Velocity-dependent damping. Mode switching driven by velocity.
Hybrid Mass-Spring

- case 1: Parameters acting on continuous dynamics.
  - CPU time approx. 140 mn!
Hybrid Mass-Spring

- case 2: parameters acting on discrete transition.
  - CPU time approx. 40 mn
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Contractors for hybrid dynamical systems
- To build upon a hybrid reachability approach

Effective methods for set membership estimation
- SM parameter estimation …
- SM hybrid state estimation of nonlinear hybrid systems

Combine with decision making for FDI
- Application to actual hybrid systems


