On feedback target control for uncertain discrete-time systems through polyhedral techniques

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Introduction

Problems of feedback target control for linear and bilinear dynamical discrete-time systems under uncertainties and state constraints are considered.

There are known approaches to solving problems of this kind, including ones for differential systems, based on construction of solvability tubes (Krasovskii's bridges).

Since practical construction of such tubes may be cumbersome, different numerical methods were devised, including methods based on approximations of sets by polytopes with a large number of vertices. Other techniques are based on estimates of sets by domains of some fixed shape such as ellipsoids and parallelepipeds. Such methods are ideologically close to interval analysis. Their main advantage is that they allow to find solutions by rather simple means. More accurate approximations may be obtained by using the whole families of such simple estimates (as proposed by A.B.Kurzhanski).

In particular, constructive computation schemes for solving the feedback target control problems for linear systems by ellipsoidal techniques were proposed and then expanded to a polyhedral technique.

Here we continue the development of polyhedral control synthesis for discrete-time systems using parallelepipeds and papallelotopes as basic sets.

Definitions of parallelepiped, parallelotope and zone

Parallelepiped in $\mathbb{R}^n$: $(P = \{p^i\} \in \mathbb{R}^{n\times n},\ \det P \neq 0)$

$$\mathcal{P} = \mathcal{P}(p, P, \pi) = \{x \mid x = p + \textstyle\sum_{i=1}^n p^i \pi_i \xi_i,\ |\xi_i| \leq 1\}.$$

Parallelotope in $\mathbb{R}^n$: $(\bar{P} = \{\bar{p}^i\} \in \mathbb{R}^{n\times r},\ r \leq n)$

$$\mathcal{P} = \mathcal{P}[p, \bar{P}] = \{x \mid x = p + \textstyle\sum_{i=1}^r \bar{p}^i \xi_i,\ |\xi_i| \leq 1\}.$$

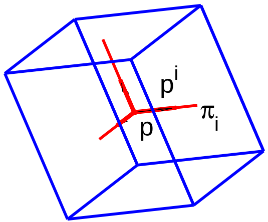
Zone: intersection of $m \leq n$ strips: $(S = \{s^i\},\ \mathrm{rank}\, S = m)$

$$\mathcal{S} = \mathcal{S}(c, S, \sigma, m) = \bigcap_{i=1}^m \Sigma^i, \quad \Sigma^i = \Sigma(c_i, s^i, \sigma_i) = \{x \,|\, |{s^i}^\top x - c_i| \leq \sigma_i\}.$$

Each parallelepiped is a parallelotope:
$\mathcal{P}(p, P, \pi) = \mathcal{P}[p, \bar{P}],\ \bar{P} = P \cdot \mathrm{diag}\, \pi$.

Each parallelotope with $r = n, \det \bar{P} \neq 0$,

and each zone with $m = n$ are parallelepipeds.



Control discrete-time systems with uncertainties

$$x[k] = (A[k] + V[k] + U[k])\,x[k-1] + B[k]u[k] + C[k]v[k],\;\; k=1,\dots,N,$$

$$x[N] \in \mathcal{M} \text{ (given target set).}$$

$$U[k] \equiv 0, \quad u[k] \text{ (controls)} \in \mathcal{R}[k] \subset \mathbb{R}^{n_u},\;\; k=1,\dots,N, \quad \text{or}$$

$$U[k] \text{ (controls)} \in \mathcal{U}[k] = \{U \in \mathbb{R}^{n\times n} \mid \mathrm{Abs}\,(U - \tilde{U}[k]) \le \hat{U}[k]\},\;\; u[k] \equiv 0.$$

$$v[k] \text{ (disrurbances)} \in \mathcal{Q}[k] \subset \mathbb{R}^{n_v},\;\; k=1,\dots,N,$$

$$V[k] \text{ (unknown matrices)} \in \mathcal{V}[k] = \{V \in \mathbb{R}^{n\times n} \mid \mathrm{Abs}\,(V - \tilde{V}[k]) \le \hat{V}[k]\}.$$

We presume:

$$\mathcal{R}[k] = \mathcal{P}[r[k], \bar{R}[k]],\;\; \mathcal{Q}[k] = \mathcal{P}[q[k], \bar{Q}[k]] \quad \text{(parallelotopes)}$$

$$\mathcal{M} = \mathcal{P}(p_\theta, P_{\mathrm{f}}, \pi_{\mathrm{f}}) = \mathcal{P}[p_{\mathrm{f}}, \bar{P}_{\mathrm{f}}] \text{ (nondegenerate parallelepiped)},\;\; \det \bar{P}_{\mathrm{f}} \neq 0.$$

For the above system we consider the following cases:

(I) without uncertainty: $V \equiv 0$, v is given (i.e., $\tilde{V} \equiv \hat{V} \equiv 0$, $\bar{Q} \equiv 0$);

(II) under uncertainty including the following two subcases:

(II,i) only additive uncertainty ($V \equiv 0$);

(II,ii) also matrix uncertainty ($V \not\equiv 0$).

$$x[k] \in \mathcal{Y}[k] \text{ (state constraints)},\;\; k=0,\dots,N-1 \quad (\mathcal{Y}[k] \text{ are zones}).$$

Problems

Problem 1 (for the case $U[k] \equiv 0$)

Let $U[k] \equiv 0$. For any i, $0 \le i \le N-1$, find a solvability set $\mathcal{W}[i]$ and a feedback control strategy $u = u[k,x]$ with $u[k,x] \in \mathcal{R}[k]$ such that each solution $x[\cdot]$ to

$$x[k]=(A[k]+V[k])x[k-1]+B[k]u[k,x[k-1]]+C[k]v[k],\ k=i+1,\dots,N,$$

that start from any $x[i] \in \mathcal{W}[i]$ would reach the target set ($x[N] \in \mathcal{M}$) and satisfy state constraints $x[k] \in \mathcal{Y}[k]$ whatever are admissible $v[\cdot]$ and $V[\cdot]$.

The function $\mathcal{W}[k]$, $k=0,\dots,N$, is called a solvability tube $\mathcal{W}[\cdot]$.

Solution for cases (I),(II,i) without matrix uncertainty (A.Vazhentsev):

$$\mathcal{W}[k-1] = A[k]^{-1}((\mathcal{W}[k] \dot{-} C[k]\mathcal{Q}[k]) - B[k]\mathcal{R}[k]) \cap \mathcal{Y}[k-1],$$
$$k = N,\dots,1; \quad \mathcal{W}[N] = \mathcal{M};$$

$$u[k,x] \in \mathcal{U}[k,x] = \mathcal{R}[k] \cap \{u \mid B[k]u \in (\mathcal{W}[k] \dot{-} C[k]\mathcal{Q}[k]) - A[k]x\}.$$

Operations with sets

We deal with following operations with sets:

Minkowski's sum: $\mathcal{X}^1 + \mathcal{X}^2 = \{y \mid y = x^1 + x^2,\ x^k \in \mathcal{X}^k\}$.

Minkowski's difference: $\mathcal{X}^1 \dot{-} \mathcal{X}^2 = \{y \mid y + \mathcal{X}^1 \subseteq \mathcal{X}^2\}$.

Intersection of sets: $\mathcal{X}^1 \cap \mathcal{X}^2$.

External (internal) polyhedral estimate $\mathcal{P}$ for $\mathcal{Q}$:

$$\mathcal{Q} \subseteq \mathcal{P} \qquad (\mathcal{P} \subseteq \mathcal{Q}).$$

Problems (continuation)

Problem 2 (for the case $U[k] \equiv 0$)

Let $U[k] \equiv 0$. Find a polyhedral tube $\mathcal{P}^-[\cdot]$ that satisfies $\mathcal{P}^-[k] \subseteq \mathcal{Y}[k]$, $k{=}0, \ldots, N{-}1$, and $\mathcal{P}^-[N] = \mathcal{M}$, and find a corresponding feedback control strategy $u = u[k, x]$ such that $u[k, x] \in \mathcal{R}[k]$ for $x \in \mathcal{P}^-[k{-}1]$, $k{=}1, \ldots, N$, and each solution $x[\cdot]$ to

$$x[k]{=}(A[k]{+}V[k])x[k{-}1]{+}B[k]u[k, x[k{-}1]]{+}C[k]v[k], k{=}1, \ldots, N,$$

with $x[0] = x_0 \in \mathcal{P}^-[0]$ would satisfy $x[k] \in \mathcal{P}^-[k]$, $k{=}1, \ldots, N$, whatever are admissible $v[\cdot]$ and $V[\cdot]$.

Introduce a family of such tubes $\mathcal{P}^-[\cdot]$.

Problems (continuation)

Problem 3 (for the case $u[k] \equiv 0$)

Let $u[k] \equiv 0$. Find a polyhedral tube $\mathcal{P}^{-}[\cdot]$ that satisfies $\mathcal{P}^{-}[k] \subseteq \mathcal{Y}[k]$, $k{=}0, \ldots, N{-}1$, and $\mathcal{P}^{-}[N] = \mathcal{M}$, and find a corresponding feedback control strategy $U = U[k, x]$ such that $U[k, x] \in \mathcal{U}[k]$ for $x \in \mathcal{P}^{-}[k{-}1]$, $k{=}1, \ldots, N$, and each solution $x[\cdot]$ to

$$x[k]{=}(A[k]{+}U[k, x[k{-}1]]{+}V[k])x[k{-}1]{+}C[k]v[k], k{=}1, \ldots, N,$$

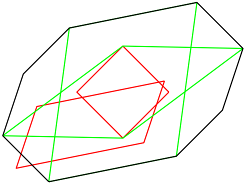
with $x[0] = x_0 \in \mathcal{P}^{-}[0]$ would satisfy $x[k] \in \mathcal{P}^{-}[k]$, $k{=}1, \ldots, N$, whatever are admissible $v[\cdot]$ and $V[\cdot]$.

Introduce a family of such tubes $\mathcal{P}^{-}[\cdot]$.

Internal estimates for Minkowski's sum $\mathcal{Q} = \mathcal{P}^1 + \mathcal{P}^2$, where $\mathcal{P}^j = \mathcal{P}[p^j, \bar{P}^j]$, $\bar{P}^1 \in \mathbb{R}^{n\times n}$, $\bar{P}^2 \in \mathbb{R}^{n\times r}$:

$$\mathbf{P}^-_\Gamma(\mathcal{Q}) = \mathcal{P}[p^1 + p^2, \bar{P}^1 + \bar{P}^2\Gamma], \quad \text{where parameter } \Gamma \in \mathcal{G}^{r\times n},$$

$$\mathcal{G}^{r\times n} = \{\Gamma \in \mathbb{R}^{r\times n} \mid \|\Gamma\| \le 1\} \qquad (\|\Gamma\| = \max_{1\le\alpha\le r} \textstyle\sum_{\beta=1}^n |\gamma_\alpha^\beta|).$$



$\mathcal{P}^1, \mathcal{P}^2$ (red),
$\mathcal{Q} = \mathcal{P}^1 + \mathcal{P}^2$ (black),
$\mathbf{P}^-_{\Gamma^i}(\mathcal{Q})$, $i = 1, 2$ (green).

Minkowski's difference $\mathcal{Q} = \mathcal{P}^1 \dot{-} \mathcal{P}^2$:

$\mathcal{Q} = \mathcal{P}[p^1 - p^2, \bar{P}^1 \operatorname{diag} \pi^*]$, if $\pi^* \ge 0$; otherwise $\mathcal{Q} = \emptyset$,
where $\pi^* = \mathrm{e} - \mathrm{Abs}\,((\bar{P}^1)^{-1}\bar{P}^2)\,\mathrm{e}$, $\quad \mathrm{e} = (1, \dots, 1)^\top$.

Primary polyhedral estimates for sets

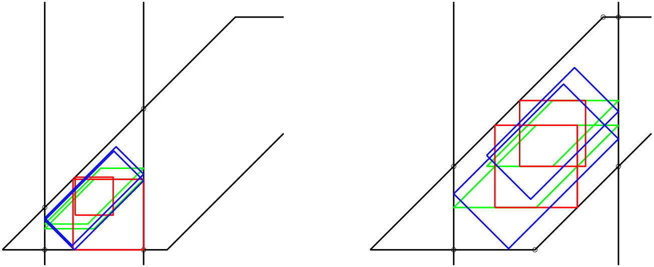
Internal estimates for $\mathcal{Q} = \bigcap_{j=1}^{\Upsilon} \Sigma^j$, where $\Upsilon \geq n+1$:

$\mathbf{P}^{-}_{p^-,P^-}(\mathcal{Q})$ can be constructed by explicit formulas for fixed parameters p^- (center) $\in \mathcal{Q}$, P^- (orientation matrix).

One of ways for calculating p^- (when P^- is fixed):

$$p^- \in \operatorname{Argmax}\{\mathrm{vol}\, \mathbf{P}^{-}_{p^-,P^-}(\mathcal{Q}) \mid p^- \in \mathcal{Q}\}$$

(using the Nelder-Mead simplex method).



Polyhedral control synthesis in Problem 2 without matrix uncertainty: way I (previous results)

Polyhedral analogue for the above relations for the solvability tube $\mathcal{W}[k-1] = A[k]^{-1}((\mathcal{W}[k] \dot{-} C[k]\mathcal{Q}[k]) - B[k]\mathcal{R}[k]) \cap \mathcal{Y}[k-1]$:

System of relations for polyhedral tubes $\mathcal{P}^-[\cdot] = \mathcal{P}[p^-[\cdot], \bar{P}^-[\cdot]]$:

$$\mathcal{P}^-[k] = \mathbf{P}^-_{p^-[k], P^-[k]}(\mathcal{P}^{0-}[k] \cap \mathcal{Y}[k]), \;\; k=N-1, \ldots, 0,$$

where $\mathcal{P}^{0-}[\cdot] = \mathcal{P}[p^{0-}[\cdot], \bar{P}^{0-}[\cdot]]$ satisfy the relations:

$$\mathcal{P}^{0-}[k-1] = A[k]^{-1}\mathbf{P}^-_{\Gamma[k]}((\mathcal{P}^-[k] \dot{-} C[k]\mathcal{Q}[k]) - B[k]\mathcal{R}[k]),$$
$$k = N, \ldots, 1; \quad \mathcal{P}^-[N] = \mathcal{M}.$$

Admissible parameters $\Gamma[\cdot]$, $P^-[\cdot]$, $p^-[\cdot]$:
such that $\quad \|\Gamma[k]\| \leq 1, \quad \det P^-[k] \neq 0, \quad p^-[k] \in \mathcal{P}^{0-}[k] \cap \mathcal{Y}[k].$

Control strategy:

$$u[k,x] \in \mathcal{U}^-[k,x] = \mathcal{R}[k] \cap \{u | \; B[k]u \in \mathcal{P}^-[k] \dot{-} C[k]\mathcal{Q}[k] - A[k]x\}.$$

System of relations for polyhedral tubes $\mathcal{P}^-[\cdot] = \mathcal{P}[p^-[\cdot], \bar{P}^-[\cdot]]$:

$$\mathcal{P}^-[k] = \mathbf{P}^-_{p^-[k],P^-[k]}(\mathcal{P}^{0-}[k] \cap \mathcal{Y}[k]), \;\; k{=}N{-}1, \ldots, 0,$$

where $\mathcal{P}^{0-}[\cdot] = \mathcal{P}[p^{0-}[\cdot], \bar{P}^{0-}[\cdot]]$ satisfy the relations:

$$p^{0-}[k{-}1]{=}D[k]^{-1}(p^-[k]{-}B[k]r[k]{-}C[k]q[k]), \;\; D[k]{=}A[k]{+}\tilde{V}[k],$$
$$\bar{P}^{0-}[k{-}1]{=}D[k]^{-1}(\bar{P}^-[k]\mathrm{diag}\,(\mathrm{e}{-}\gamma[k]{-}\beta[k]){-}B[k]\bar{R}[k]\Gamma[k]),$$
$$\gamma[k] = (\mathrm{Abs}\,(\bar{P}^-[k]^{-1}C[k]\bar{Q}[k]))\mathrm{e},$$
$$\beta[k]{=}\max_{z\in\mathbb{E}(\mathcal{P}^{0-}[k-1])}(\mathrm{Abs}\,(\bar{P}^-[k]^{-1}))\hat{V}[k]\,\mathrm{Abs}\,z$$

(where $\mathbb{E}(\mathcal{P})$ denote vertices of $\mathcal{P}$), $k{=}N, \ldots, 1$; $p^-[N]{=}p_\mathrm{f}$, $\bar{P}^-[N]{=}\bar{P}_\mathrm{f}$.

In fact, $\beta[k]$ satisfies the system of equations: $\beta[k] = H[k, \beta[k]]$.
For cases (I), (II,i) (i.e., without matrix uncertainty) $\beta[k]{=}0$.

Admissible parameters $\Gamma[\cdot]$, $P^-[\cdot]$, $p^-[\cdot]$:
such that $\|\Gamma[k]\| \leq 1$, $\det P^-[k] \neq 0$, $p^-[k] \in \mathcal{P}^{0-}[k] \cap \mathcal{Y}[k]$.

Control strategy:

$$u[k,x] = r[k] + \bar{R}[k]\Gamma[k]\bar{P}^{0-}[k-1]^{-1}(x - p^{0-}[k-1]), \quad k=1,\ldots,N.$$

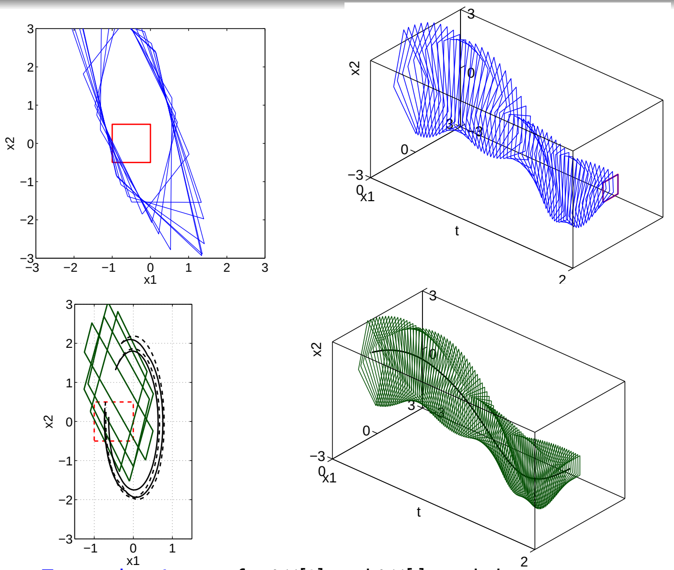
Theorem 1

Let $\Gamma[\cdot]$, $P^{-}[\cdot]$, $p^{-}[\cdot]$ be arbitrary admissible parameters and the above system has a solution $(p^{0-}[\cdot], \bar{P}^{0-}[\cdot], p^{-}[\cdot], \bar{P}^{-}[\cdot])$ such that we obtain $\mathrm{e} - \gamma[k] - \beta[k] > 0$, $\det \bar{P}^{-}[k] \neq 0$ for $k=N,\ldots,1$. Then $\mathcal{P}^{-}[\cdot]$ and $u[\cdot,\cdot]$ give a solution to Problem 2.

If $\|H[k,\beta^1] - H[k,\beta^2]\| \leq L[k]\|\beta^1 - \beta^2\|_\infty$, where $L[k] \in (0,1)$, then the equation $\beta = H[k,\beta]$ has a solution $\beta = \beta[k] \geq 0$, which can be found by the iteration $\beta^{l+1} = H[k,\beta^l]$, $l=0,1,\ldots,$ $\beta^0 = 0$.

Let the discrete-time system is obtained by the Euler approximations of some differential equations: $A[k]=I+h_N A(t_{k-1})$, $B[k]=h_N B(t_{k-1})$, $\mathcal{R}[k]=\mathcal{R}(t_{k-1})$, …, $t_k=kh_N\in[0,\theta]$, $h_N=\theta N^{-1}$. Then we have $L[k]<1$ and $\mathrm{e}-\gamma[k]-\beta[k] > 0$ for a fixed k if $\det \bar{P}^{-}[k]\neq 0$ and h_N is sufficient small.

Example 1: (without uncertainty and state constr.), n=2

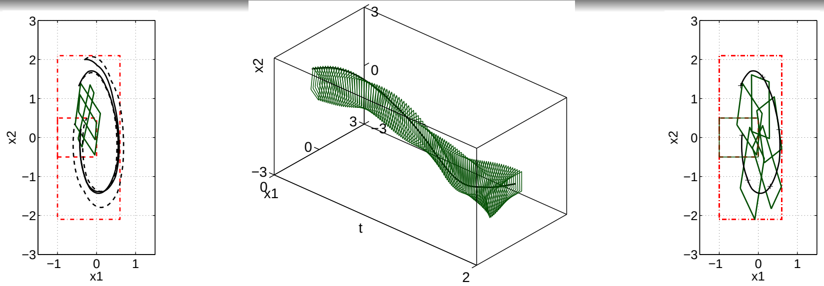


External estimates for $\mathcal{W}[0]$ and $\mathcal{W}[\cdot]$, and the target set.

Several cross-sections $\mathcal{P}^-[0]$ and some tube $\mathcal{P}^-[\cdot]$

(they also are internal estimates for $\mathcal{W}[0]$, $\mathcal{W}[\cdot]$), and controlled trajectories.

Example 1: case (II,ii) with state constraints, n=2



Several cross-sections $\mathcal{P}^{-}[0]$, some polyhedral tube $\mathcal{P}^{-}[\cdot]$, several cross-sections $\mathcal{P}^{-}[k]$ for this $\mathcal{P}^{-}[\cdot]$, and controlled trajectories.

Data:

$A \equiv I + \tau \begin{bmatrix} 0 & 1 \\ -8 & 0 \end{bmatrix}$, $\tilde{V} \equiv 0$, $\hat{V} \equiv \tau \cdot \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}$, $B \equiv C \equiv \tau \cdot I$,
$\mathcal{R} \equiv \mathcal{P}(0, I, (0,1)^{\top})$, $\mathcal{M} = \mathcal{P}((-0.5,0)^{\top}, I, (0.5,0.5)^{\top})$,
$\mathcal{Q} \equiv \mathcal{P}(0, I, (0.2,0)^{\top})$, $\tau = \theta/N$, $\theta = 2$, N=200.
State constraints: $|x_1 + 0.2| \leq 0.8$, $|x_2| \leq 2.1$.

Polyhedral control synthesis in Problem 3

System of relations for polyhedral tubes $\mathcal{P}^-[\cdot] = \mathcal{P}[p^-[\cdot], \bar{P}^-[\cdot]]$:

$$\mathcal{P}^-[k] = \mathbf{P}^-_{p^-[k],P^-[k]}(\mathcal{P}^{0-}[k] \cap \mathcal{Y}[k]),\ \ k{=}N{-}1,\dots,0,$$

where $\mathcal{P}^{0-}[\cdot] = \mathcal{P}[p^{0-}[\cdot], \bar{P}^{0-}[\cdot]]$ satisfy the relations:

$$p^{0-}[k{-}1]{=}D[k]^{-1}(p^-[k]{-}C[k]q[k]),\ \ D[k]{=}A[k]{+}\tilde{U}[k]{+}\tilde{V}[k],$$

$$\bar{P}^{0-}[k{-}1] = H[k, \bar{P}^{0-}[k{-}1]], \quad k = N, \dots, 1,$$

$$H[k,P] = (D[k] - \mathrm{diag}\,\alpha[k,P;J[k]])^{-1}\,\bar{P}^-[k]\,\mathrm{diag}\,(\mathrm{e} - \beta[k,P] - \gamma[k]),$$

$\alpha[k,P;J[k]],\ \beta[k,P],\ \gamma[k]$ are given by explicit formulas.

$$p^-[N]{=}p_{\mathrm{f}},\ \ \bar{P}^-[N]{=}\bar{P}_{\mathrm{f}}.$$

Here $P = \bar{P}^{0-}[k{-}1]$ satisfies the system of equations $P = H[k,P]$.
Admissible parameters: $J[\cdot]$, where $J[k]{=}\{j_1[k],\dots,j_n[k]\}$ are arbitrary permutations of numbers $\{1,\dots,n\}$,
and $P^-[\cdot]$, $p^-[\cdot]$ such that $\det P^-[k] \neq 0$, $p^-[k] \in \mathcal{P}^{0-}[k] \cap \mathcal{Y}[k]$.

Control strategy:

$$U[k,x]\,x{=}\tilde{U}[k]\,x{-}\mathrm{diag}\,\alpha[k,\bar{P}^{0-}[k{-}1];J[k]]\,(x{-}p^{0-}[k{-}1]),\ \ k{=}1,\dots,N.$$

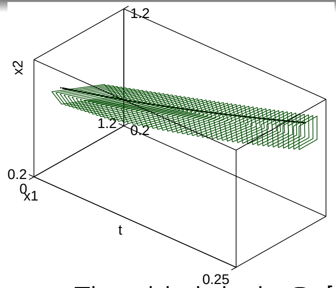
Theorem 2

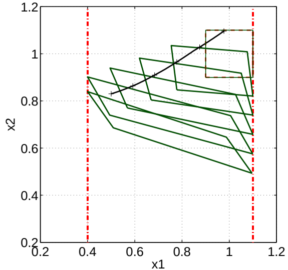
Let the above system has a solution $(p^{0-}[\cdot], \bar{P}^{0-}[\cdot], p^{-}[\cdot], \bar{P}^{-}[\cdot])$ such that we obtain $\det \bar{P}^{-}[k] \neq 0$ and $\mathrm{e} - \beta[k, \bar{P}^{-}[k-1]] - \gamma[k] \geq 0$ for $k = N, \ldots, 1$. Then $\mathcal{P}^{-}[\cdot]$ and $U[\cdot, \cdot]$ give a particular solution to Problem 3.

Let the discrete-time system be obtained by the Euler approximations of some differential equations. Let, for a fixed k, $\det \bar{P}^{-}[k] \neq 0$ and the time step h_N be sufficient small.

Then the above operator $H[k, P]$ is contractive, and, therefore, the equation $P = H[k, P]$ has a solution $P = \bar{P}^{0-}[k-1]$, which can be found by the simple iteration $P^{l+1} = H[k, P^{l}]$, $l = 0, 1, \ldots$, starting from $P^0 = \bar{P}^{-}[k]$. Also, we have $\mathrm{e} - \beta[k, \bar{P}^{0-}[k-1]] - \gamma[k] > 0$.

Example 2: case (II,ii) with state constraints, n=2





The polyhedral tube $\mathcal{P}^-[\cdot]$, several cross-sections $\mathcal{P}^-[k]$ for $\mathcal{P}^-[\cdot]$, and the controlled trajectory.

Data:

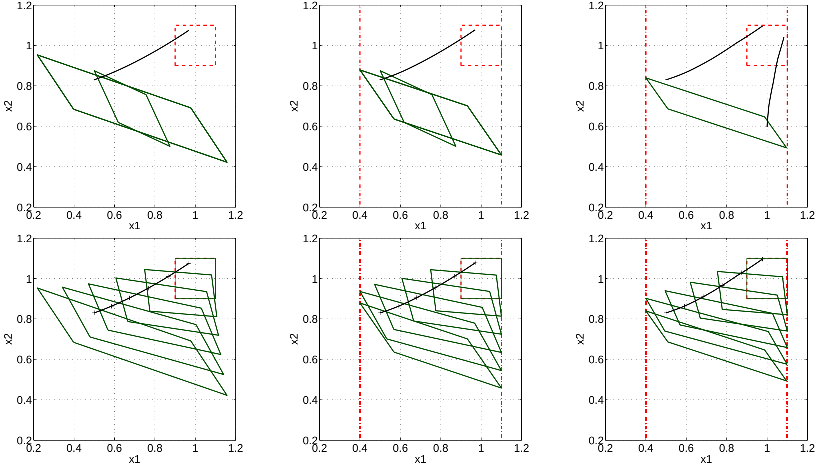
$$A \equiv I + \tau \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \quad \tilde{U} \equiv \tau \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}, \hat{U} \equiv \tau \begin{bmatrix} 0 & 1.5 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{V} \equiv \tau \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, \hat{V} \equiv \tau \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}, \mathcal{Q} \equiv \mathcal{P}(0, I, (0.05, 0)^\top), \ C \equiv \tau \cdot I,$$

$$\mathcal{M} = \mathcal{P}((1,1)^\top, I, (0.1, 0.1)^\top), \quad \tau = \theta / N, \quad \theta = 0.25, \quad N{=}200.$$

State constraints: $|x_1 - 0.75| \leq 0.35$

Example 2: (under uncertainties and state constraints), n=2



Case (I) without state constraints (SC); case (I) with SC; case (II,ii) with SC.
Cross-sections $\mathcal{P}^-[0]$ and controlled trajectories (top).
Several cross-sections $\mathcal{P}^-[k]$ and the controlled trajectories (bottom).

Conclusion

Two types of problems of feedback terminal target control for linear and bilinear discrete-time uncertain systems under state constraints are considered, where controls appear either additively or in the system matrix.

- Polyhedral control synthesis using polyhedral (parallelotope-valued) solvability tubes is elaborated.
  The cases without uncertainties, with additive uncertainties, and also with a matrix uncertainty are considered.
- Nonlinear recurrent relations are presented for polyhedral solvability tubes.
- Control strategies, which can be calculated by explicit formulas on the base of these tubes, are proposed.
- The results of numerical simulations are presented.
  Proposed polyhedral solvability tubes may turn out to be rather conservative. But we can easily calculate them, while it is hard to calculate maximal solvability tubes.

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