Rigorous computation of Poincaré maps

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Introduction

There are many powerful topological and smooth tools for studying dynamics of maps. These are, in particular, the Interval Newton Operator, Brouwer degree, covering relations, cone conditions and many other.

In continuous time dynamical systems there are some tools that can be used for validation of some properties without integration of the system - for instance isolating blocks, Conley index and isolating segments. It turns out, however, that using the concept of Poincaré map combined with efficient ODE/PDE solver one can apply the well known tools available for maps for studying continuous-time dynamics. The applicability of this concept is proved by many examples, just to mention few of them:

- chaos in the Lorenz, Rössler systems and Planar Circular Restricted Three Body problem,
- the existence of choreographic solutions to the n-body problem,
- heteroclinic and homoclinic dynamics,
- various local bifurcations (like period doubling, homoclninic tangencies) and global bifurcations (cocoon bifurcations, Shilnikov homoclinic bifurcations),

- the existence and uniform hyperbolicity of attractors,
- periodic solutions to delay and partial differential equations.

All the above results have been obtained by the authors by means of Poincaré map techniques and ODE solvers. Therefore it is very important to have a good numerical algorithms for rigorous computation of Poincaré maps.

The talk will be based on our over 20 years of experience in designing and implementation of rigorous ODE solvers and interval tools for Poincaré maps. The software is available as a part of the CAPD library.

Main results

It turns out that obtained enclosures for Poincaré map can be significantly reduced by proper choice of Poincaré section. Even in the case when the Poincaré section cannot be changed (for instance due to some engineering reasons or presence of symmetries which should be preserved) we can still manipulate coordinate system in which we represent the arguments and values of Poincaré maps.

In the talk we will show that the optimal choice of Poincaré section nearby periodic orbit is related to left eigenvectors of the derivative of Poincaré map at a periodic point.

We will also give an algorithm for efficient computation of Poincaré map in a given coordinate system that takes into account internal representation of solutions in an ODE solver.

These heuristics are confirmed by tests we performed.

References

[**1**] CAPD - `http://capd.ii.uj.edu.pl` - the CAPD library, a C++ toolbox for rigorous dynamics.

[**2**] T. Kapela, D. Wilczak and P. Zgliczyński, Rigorous computation of Poincaré maps, *in preparation.*