Cooperative Localization And Formation Maintaining Using Range-only Measurements Without Communications

Mohamed Saad IBN SEDDIK, Luc JAULIN and Jonathan GRIMSDALE

April 30, 2015

Abstract

This paper is about using range-only measurements between multiple Autonomous Underwater Vehicles (AUV) to maintain a predefined formation. The AUVs have no a priori knowledge of each other's path or decisions. All vehicles must maintain the same speed during the mission and are not allowed to stop. Each vehicle must then adjust its position using the available range-only data. We provide a guaranteed state estimation of the targeted vehicle using interval analysis and a set-membership approach. Simulation results are described and discussed.

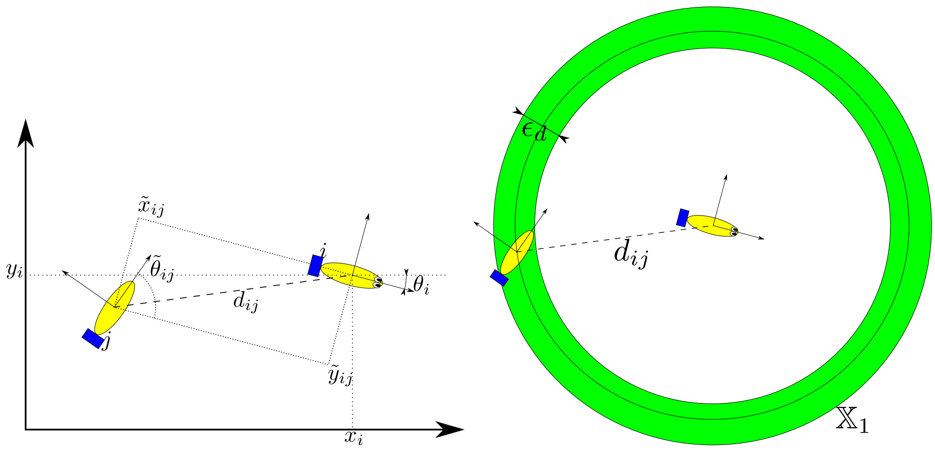
1 Introduction And Background

Envision a scenario where a swarm of Autonomous Underwater Vehicles (AUV) is exploring an area. Due to the large number of vehicles and because of the communication medium (underwater acoustics), the vehicles can hardly exchange their position with their neighbors to maintain a given formation. In this paper, we propose to use range-only measurements to keep the formation. Using synchronized clocks, all vehicles emit a unique ping at a given time. The one-way acoustic time of flight is used to determine the range between vehicles. One-Way time of flight has already been explored using stationary objects [3] and a similar study has been done with communicating moving AUVs [1, 7].

As all the vehicles are similar and evolve at the same depth, the state of each vehicle i will be represented with its position in a 2-D plane and its heading, $\mathbf{x}_i = [x_i, y_i, \theta_i]^T$. For the mission purposes, the vehicles must keep a given speed v and can only control their rotation speed $u_i \in U$, where U is the set of all possible rotation speeds. For the sake of simplicity, let the robot motion be described by the following state equations

$$\begin{cases} \dot{x}_i &= v_i \cdot \cos\theta_i \\ \dot{y}_i &= v_i \cdot \sin\theta_i \\ \dot{\theta}_i &= u_i \end{cases} \tag{1.1}$$

Due to the lack of information about the vehicles' absolute position, we propose to study the relative position and therefore a relative localization. Let f be the evolution function of the AUV j's relative position $\tilde{\mathbf{x}}_{ij}$ in the vehicle



(a) Representation of two vehicles of the swarm.

(b) With range-only information at the first step, $\mathbb{X}_1$ is a cylinder (projected here on the 2-D space).

Figure 1.1: Relative Localization

i frame, figure 1.1a. The ranging information between the two vehicles will be represented as

$$\mathbf{y}_{ij} = d_{ij} + \epsilon_d = \sqrt{\tilde{x}_{ij}^2 + \tilde{y}_{ij}^2} + \epsilon_d = g(\tilde{\mathbf{x}}_{ij}) \tag{1.2}$$

The system can then be represented in the frequently used from,

$$\Sigma : \begin{cases} \dot{\tilde{\mathbf{x}}}_{ij} & = f(\tilde{\mathbf{x}}_{ij}, u_j, u_i) \\ \mathbf{y}_{ij} & = g(\tilde{\mathbf{x}}_{ij}) \end{cases} \tag{1.3}$$

To solve the system Σ of equation (1.3), we propose to use set-membership techniques and interval analysis [5].

2 Set-Membership Estimator

Measurements of distance come at discrete instants. Let us consider the system in equation (1.3) at a discrete time domain using Euler's discretization:

$$\Sigma : \begin{cases} \tilde{\mathbf{x}}_{ij}^k & = \tilde{f}(\tilde{\mathbf{x}}_{ij}^{k-1}, u_j^{k-1}, u_i^{k-1}) \\ \mathbf{y}_{ij}^k & = g(\tilde{\mathbf{x}}_{ij}^k) \end{cases}, k = 0, \dots, t_{final} \tag{2.1}$$

where $\tilde{\mathbf{x}}_{ij}^k \in \mathbb{R}^3$ is the state vector at the discrete time k , $\mathbf{y}_{ij}^k \in \mathbb{R}$ is the output and $\tilde{f}$ is the Euler integral of f.

Let $\mathbb{X}_k$ be the associated domain set of the variable $\tilde{\mathbf{x}}^k$[1] at the instant k. As no prior information is available on $\tilde{\mathbf{x}}^0, \dots, \tilde{\mathbf{x}}^{t_{final}}$, thus $\mathbb{X}_0, \dots, \mathbb{X}_{t_{final}}$ are taken as $\mathbb{R}^3$. The measurement $\mathbf{y}^k$, with the noise value ϵ_d, is used to form the

[1] For simplification purposes, the indexes ij will be omitted.

measurement set $\mathbb{Y}_k$. u_i is known as the robot's own input, and we define $\mathbb{U}_k$ is the set of all possible u_j^k. The equation (2.1) then becomes

$$\begin{cases} \mathbb{X}_k &= \tilde{f}(\mathbb{X}_{k-1}, \mathbb{U}_{k-1}, u_i^{k-1}) \\ \mathbb{Y}_k &= g(\mathbb{X}_k) \end{cases} \tag{2.2}$$

At every instant k, the state set $\mathbb{X}_k$ is defined with equation (2.2), thus it can be computed with

$$\mathbb{X}_k = \tilde{f}(\mathbb{X}_{k-1}, \mathbb{U}_{k-1}, u_i^{k-1}) \cap g^{-1}(\mathbb{Y}_k),\ k = 1, \ldots, t_{final} \tag{2.3}$$

Let's take as an example the first two set $\mathbb{X}_0$ and $\mathbb{X}_1$. As no prior information is available, $\mathbb{X}_0 = \mathbb{X}_1 = \mathbb{IR}^3$. Then applying equation (2.3) to $\mathbb{X}_1$ will contract the space to a cylinder in $\mathbb{IR}^{\mathbf{3}}$, see figure 1.1b.

Reversibly, $\mathbb{X}_{k-1}$ can be computed from the recently computed $\mathbb{X}_k$ using the inverse of $\tilde{f}$,

$$\mathbb{X}_{k-1} = \tilde{f}^{-1}(\mathbb{X}_k, \mathbb{U}_{k-1}, u_i^{k-1}),\ k = 1, \ldots, t_{final} \tag{2.4}$$

Applying both equation (2.3) and equation (2.4) to every set $\mathbb{X}_k$ is similar to applying a non-causal state estimator [4].

3 Preliminary Results

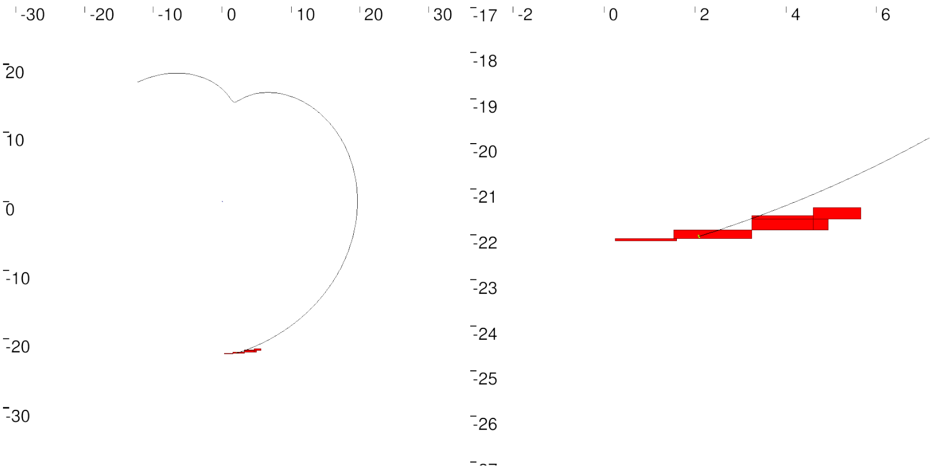
A scenario of two vehicles is simulated, where, first, a vehicle i loiters around a position and the vehicle j moves in a straight line.

Applying the algorithm on the data set provided by the simulation, figure 3.1, shows that it provides a relatively accurate position estimation of the targeted vehicle. In this figure, the red boxes represent a projection of the set $\mathbb{X}_k$ of the (xOy) plane. One will notice that the set $\mathbb{X}_k$ is made of multiple boxes, this is due to the bisection algorithm [5, 6].

As the vehicle i has a relatively accurate position of the position of the vehicle j, it can now maintain the formation by going to a desired position knowing the set $\mathbb{X}_k$ where the other vehicle is. A strategy based behavior [2] can be implemented to keep the formation as this algorithm is symmetrical. For example, when j realizes that it is too far from i and knows that the latter will not be able to maintain the formation because of the speed limitation, it can loiter waiting for i to catch up.

4 Conclusion And Future Work

In this paper, a relative positioning based only on range information has been developed. Even though the system is non-linear and non observable, we showed that the set-membership approach can produce a relatively accurate relative position estimation based only on ranging measurements. The resulting position estimation can be enough to maintain a given formation between the vehicles. Future work will consist of simulating a swarm of AUVs all running the same algorithm. A real-life experiment will also conducted to provide realistic data to be studied as the modeling used for simulation is not as accurate.



(a) The black line represents the trajectory of j in i's frame.

(b) Zooming around the real position (represented with the yellow ellipse) shows that it is closely surrounded by the computed set $\mathbb{X}_{t_{final}}$.

Figure 3.1: Set-Membership Inversion results.

References

[1] P Baccou, B Jouvencel, V Creuze, and C Rabaud. Cooperative positioning and navigation for multiple auv operations. In *OCEANS, 2001. MTS/IEEE Conference and Exhibition*, volume 3, pages 1816–1821. IEEE, 2001.

[2] Michael R Benjamin, Joseph A Curcio, John J Leonard, and Paul M Newman. Navigation of unmanned marine vehicles in accordance with the rules of the road. In *Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on*, pages 3581–3587. IEEE, 2006.

[3] Ryan M Eustice, Louis L Whitcomb, Hanumant Singh, and Matthew Grund. Experimental results in synchronous-clock one-way-travel-time acoustic navigation for autonomous underwater vehicles. In *Robotics and Automation, 2007 IEEE International Conference on*, pages 4257–4264. IEEE, 2007.

[4] L. Jaulin, M. Kieffer, I. Braems, and E. Walter. Guaranteed nonlinear estimation using constraint propagation on sets. *International Journal of Control*, 74(18):1772–1782, 2001.

[5] L. Jaulin and E. Walter. Set inversion via interval analysis for nonlinear bounded-error estimation. *Automatica*, 29(4):1053–1064, 1993.

[6] Mohamed Saad Ibn Seddik, Luc Jaulin, and Jonathan Grimsdale. Phase based localization for underwater vehicles using interval analysis. *Mathematics in Computer Science*, 8(3-4):495–502, 2014.

[7] Xun S Zhou and Stergios I Roumeliotis. Robot-to-robot relative pose estimation from range measurements. *Robotics, IEEE Transactions on*, 24(6):1379–1393, 2008.