

Goal programming approach for solving interval MOLP problems

S. Rivaz¹, M. A. Yaghoobi² and M. Hladík³

¹ Department of Mathematics, Yasouj University, Yasouj, Iran
sanazrivaz@gmail.com

² Department of Applied Mathematics,
Shahid Bahonar University of Kerman, Kerman, Iran
yaghoobi@uk.ac.ir

³ Department of Applied Mathematics,
Charles University, Malostranské nám. 25, 11800 Prague, Czech Republic
hladik@kam.mff.cuni.cz

Keywords: Multiobjective linear programming, interval programming, goal programming

Introduction

Uncertainty of the model parameters are frequently involved in most of the decision-making problems due to inherent imprecise in nature of human judgments.

To deal with such imprecisions, various ways are existed. One of them is interval based approach in which the ambiguity of parameters are modeled by intervals. In this article, multiobjective linear programming problems with interval objective functions coefficients, or interval MOLP problems for short, are investigated. Such problems have gained many researchers' interests in the last three decades; an overview on interval MOLP was given by Oliveira et al. [4].

It is well known that goal programming method is one of the most popular and powerful methods in MOLP [2]. In the context of goal programming, Inuiguchi and Kume have derived four formulations for an interval MOLP problem with target intervals [1]. It should be noted that since the decision-maker is not always an expert, he/she cannot decide which formulation is the most appropriate one for his/her situation.

In this paper, we use a specified distance concept between intervals to introduce a unique model for an interval MOLP problem with target intervals, via goal programming approach. This model is derived from a simple mathematical procedure and because of its uniqueness, the decision-maker has no doubt about its appropriateness.

Problem statement and the proposed method

We consider the following interval MOLP problem with interval targets:

Optimize:

$$\sum_{j=1}^n c_{kj}x_j = t_k, \quad k = 1, \dots, p, \quad (1)$$

$$s.t. \quad Ax \leq b,$$

$$c_{kj} \in C_{kj}, \quad k = 1, \dots, p, \quad j = 1, \dots, n,$$

$$t_k \in T_k, \quad k = 1, \dots, p,$$

$$x \geq 0,$$

where C_{kj} is the closed interval $[c_{kj}^l, c_{kj}^u]$ representing a region the coefficients c_{kj} possibly take. T_k is the closed interval $[t_k^l, t_k^u]$ representing a region the target value t_k possibly take. The constraints of the problem are as in conventional MOLP problem. our aim, for solving problem (1), is to bring each planned interval $[\sum_{j=1}^n c_{kj}^l x_j, \sum_{j=1}^n c_{kj}^u x_j]$ as close as possible to target interval $[t_k^l, t_k^u]$, $k = 1, \dots, p$. It means that minimizing the total distance of planned intervals from target intervals is needed. In order to do that, we use the following distance concept between two intervals.

Definition 1. ([3]) Let $[a^l, a^u]$ and $[b^l, b^u]$ be two intervals. Then, the distance between them is defined as:

$$d([a^l, a^u], [b^l, b^u]) = \max\{|a^l - b^l|, |a^u - b^u|\}.$$

It is easy to check that the distance d , defined in above introduces a metric on the space of closed intervals in \mathbb{R} . Based on this concept, we have:

$$D_k = d([\sum_{j=1}^n c_{kj}^l x_j, \sum_{j=1}^n c_{kj}^u x_j], [t_k^l, t_k^u]) = \\ \max(|\sum_{j=1}^n c_{kj}^l x_j - t_k^l|, |\sum_{j=1}^n c_{kj}^u x_j - t_k^u|), k = 1, \dots, p,$$

as the distance between the k th planned interval and its target interval ($k = 1, \dots, p$). Using the deviational variables d_k^{l-} , d_k^{l+} , d_k^{u-} and d_k^{u+} as:

$$\sum_{j=1}^n c_{kj}^l x_j + d_k^{l-} - d_k^{l+} = t_k^l; \quad \sum_{j=1}^n c_{kj}^u x_j + d_k^{u-} - d_k^{u+} = t_k^u;$$

D_k can be written as follows:

$$D_k = \max(|d_k^{l-} - d_k^{l+}|, |d_k^{u-} - d_k^{u+}|), k = 1, \dots, p.$$

For aggregating all distances, $D(x)$ is considered:

$$D(x) = \lambda \sum_{k=1}^p w_k D_k + (1 - \lambda) \bigvee_{k=1}^p D_k,$$

where $0 \leq \lambda \leq 1$, $w_k \geq 0$, $k = 1, \dots, p$, and $\sum_{k=1}^p w_k = 1$. Now, our model, for solving an interval MOLP problem with target intervals, can be stated in the following way:

$$\min \lambda \sum_{k=1}^p w_k v_k + (1 - \lambda)u \quad (2)$$

$$\begin{aligned}
s.t. \quad & \sum_{j=1}^n c_{kj}^l x_j + d_k^{l-} - d_k^{l+} = t_k^l, \quad k = 1, \dots, p, \\
& \sum_{j=1}^n c_{kj}^u x_j + d_k^{u-} - d_k^{u+} = t_k^u, \quad k = 1, \dots, p, \\
& Ax \leq b, \\
& d_k^{l-} + d_k^{l+} \leq v_k, \quad k = 1, \dots, p, \\
& d_k^{u-} + d_k^{u+} \leq v_k, \quad k = 1, \dots, p, \\
& v_k \leq u, \quad k = 1, \dots, p, \\
& x_j \geq 0, \quad j = 1, \dots, n, \\
& v_k \geq 0, \quad k = 1, \dots, p, \\
& d_k^{l-}, d_k^{l+}, d_k^{u-}, d_k^{u+} \geq 0, \quad k = 1, \dots, p, \\
& u \geq 0.
\end{aligned}$$

Lemma 1. *Let model (2) be feasible then it has an optimal solution in which the constraints $d_k^{l-} \cdot d_k^{l+} = 0$ and $d_k^{u-} \cdot d_k^{u+} = 0$, $k = 1, \dots, p$, are satisfied.*

References

- [1] M. INUIGUCHI AND Y. KUME, Goal programming problems with interval coefficients and target intervals, *European Journal of Operational Research* 52: 345-360, 1991.
- [2] D.F. JONES AND M. TAMIZ, *Practical Goal Programming*, Springer, 2010.
- [3] R.E. MOORE, R.B. KEARFOTT AND M.J. CLOUD, *Introduction to Interval Analysis*, Society for Industrial and Applied Mathematics, Philadelphia, 2009.
- [4] C. OLIVEIRA AND C.H. ANTUNES, Multiple objective linear programming models with interval coefficients - an illustrated overview, *European Journal of Operational Research* 181: 1434-1463, 2007.