Computing Barriers of Ordinary Differential Equations

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Consider the following problem: Given an ordinary differential equation, a set of initial states, and a set of states considered to be unsafe, compute a set that

- contains all initial states,
- cannot be left by any trajectory of the differential equation, and
- does not contain any unsafe state.

Such a set is often called a barrier. It certifies that the given ODE does not have a trajectory that starts in an initial state and ends in an unsafe state.

After using an inequality $p(x_1, \ldots, x_n) \leq 0$ for representing the barrier $\{x_1, \ldots, x_n \mid p(x_1, \ldots, x_n) \leq 0\}$, and assuming that that the set of initial states, and the set of unsafe states is also given as an inequality, the three conditions mentioned above can be formulated as universally quantified constraints [4]. Moreover, after introducing parameter values $a_1, \ldots, a_k$ into the inequality representing the barrier, arriving at $p(a_1, \ldots, a_k, x_1, \ldots, x_n) \leq 0$, the problem reduces to finding parameter values $a_1, \ldots, a_k$ such that three universally quantified constraints are fulfilled. Hence, we have a problem of solving quantified constraints [5, 3, 1] with quantifier prefix $\exists\forall$.

In our talk, we discuss a variant of the above problem where we want a quick but incomplete test for the existence of certain simple barriers which is useful as a part of more complicated safety verification algorithms. We present an algorithm that uses interval techniques for handling the universal quantifier, but searches for the parameter values $a_1, \ldots, a_k$ using classical numerical techniques. We also show the results of first numerical experiments.

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