Verified Convex Hull for Inexact Data

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Introduction

This talk is concerned with a convex hull in two-dimensions. The convex hull is one of well-known topics in computational geometry. If floating-point arithmetic [1] is used for convex hull algorithm, then an inexact result may be obtained due to accumulation of rounding errors. This problem is called robustness problem and is introduced in detail in [2]. We focus on the convex hull for uncertain data, namely, data is given by set of intervals. We developed a verified algorithm for the convex hull for interval data.

Proposed Method

Three points $A = (a_x, a_y)$, $B = (b_x, b_y)$ and $C = (c_x, c_y)$ are given, where $|\tilde{a}_x - a_x| \leq r_{ax}, \ldots, |\tilde{c}_y - c_y| \leq r_{cy}$. Let a computed result D be defined as

$$D := \mathrm{fl}((\tilde{a}_x - \tilde{c}_x)(\tilde{b}_y - \tilde{c}_y) - (\tilde{a}_y - \tilde{c}_y)(\tilde{b}_x - \tilde{c}_x)),$$

where $\mathrm{fl}(\cdot)$ means that all operations inside the parentheses are evaluated by floating-point arithmetic. We developed floating-point filters

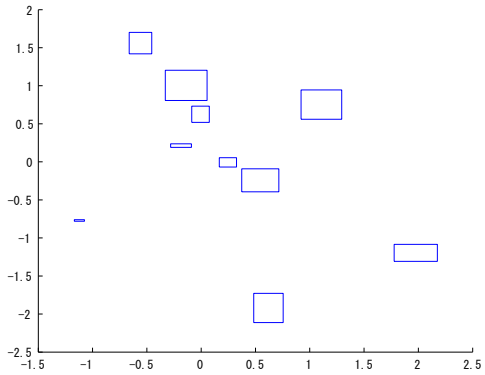

Figure 1: Input interval data

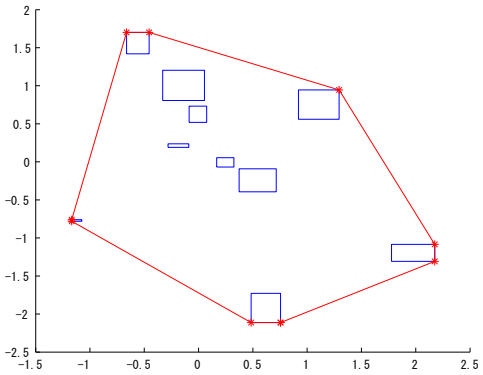

Figure 2: Outer convex hull

for a two-dimensional orientation problem with interval data. Our filters give a sufficient condition of the following

$$\text{sign}(D) := \text{sign}((a_x - c_x)(b_y - c_y) - (a_y - c_y)(b_x - c_x)), \quad \text{where } \forall a_x, \ldots, c_y.$$

Next, we developed an iterative convex hull algorithm based on the incremental algorithm for interval data. Our algorithm produces an outer convex hull [3] which encloses all intervals. Figure 1 shows input data and Figure 2 shows the outer convex hull for the input data. The details of the iterative algorithm with our floating-point filters and numerical results will be shown in the presentation.

References

[1] IEEE Standard for Floating-Point Arithmetic, Std 754-2008, 2008.

[2] L. Kettner, K. Mehlhorn, S. Pion, S. Schirra, C. Yap, Classroom Examples of Robustness Problems in Geometric Computation, *Computational Geometry*, 40:61–78, 2007.

[3] T. Nagai, S. Yasutome and N. Tokura, Convex hull problem with imprecise input, *Discrete and Computational Geometry*, Springer, 1763:207–219, 2000.