Gaussian Nonlinear set inversion

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Introduction

In this presentation, we treat the problem of estimating the parameters of a nonlinear model from experimental data in a reliable and precise manner. Using interval analysis, we are able to compute the set of all the parameters that are consistent with a given probability with the experimental data. Using statistical properties of the uncertainties associated with each measurement, we will show that a geometrical constraint can be extracted that enables us to drasticaly reduce size of the solution set.

Description

Let $\tilde{\mathbf{y}} \in \mathbb{R}^n$ be the vector of all the collected data, and $\mathbf{p} \in \mathbb{R}^m$ the parameters we want to estimate which parametrize a function $\mathbf{f}(\mathbf{p})$.

In the context of a bounded-error model, each measurement $\widetilde{y_i}$ is associated to an interval $[y_i]$ which is assumed to contain the true value y_i, and the vector of intervals $[\mathbf{y}]$ defines the set $\mathbb{Y}$ which is an n-dimensional axis-aligned box.

Then, the problem to be solved is characterizing the set $\mathbb{S}$ of all values of $\mathbf{p}$ consistent with the data:

$$\mathbb{S} = \{\mathbf{p} \in \mathbb{R}^m | \mathbf{f}(\mathbf{p}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y})$$

which is a set-inversion problem and can be efficiently solved using interval analysis [1],[2].

These methods are reliable, in the sense that no approximation, no linearization is made on the model, and guarantees that not a single feasible solution will be lost.

However the bounded-error assumption, while compatible with an infinity of probability distribution, doesn't take into acount the statistical properties of the perturbation on each measurements y_i. In [3], a method is proposed that allows to compute the set of all the parameters that are consistent with a given probability with a set of measurements, while taking into account the statistical properties of the perturbation.

We will study the widespread case where each measurement y_i is subject to a normaly distributed perturbation w_i, that is: $\widetilde{y_i} = y_i + w_i$.

In this case, the set $\mathbb{Y}$ is not a box anymore, but an n-dimensional ellipsoid which expression is given by:

$$\mathbb{Y} = \{\mathbf{y} | (\tilde{\mathbf{y}} - \mathbf{y})\mathbf{Q}^{-1}(\tilde{\mathbf{y}} - \mathbf{y})^T \leq \alpha(\eta)\}$$

with $\alpha(\eta)$ a confidence threshold depending on a given probability η of $\widetilde{\mathbf{y}}$ being in $\mathbb{Y}$, and $\mathbf{Q}$ the covariance matrix of the random vector $\mathbf{w}$.

The set-inversion problem now consists of inverting an ellipsoid, whose volume, as will be shown, is much smaller than the volume of its n-dimensional box counterpart.

As will be presented, this additional constraint will greatly enhance the precision of the estimation, in the sense that the set $\mathbb{S}$ will be much smaller.

The improvements will be presented on some test-cases.

References

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