Presentation of a multithreaded interval solver for nonlinear systems

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Introduction

The paper presents the interval branch-and-prune solver for nonlinear underdetermined and well-determined systems. The system has been developed by the author. Its various aspects have been presented in a series of publications, i.a., [3]–[7].

Narrowing operators

Several tools are incorporated in our solver to process the boxes in the branch-and-prune process. They include:

- two versions of the interval Newton operator (componentwise Newton and Gauss-Seidel; see [4]),
- enforcing two versions of consistency – box-consistency and bound-consistency; see [7],
- an initial exclusion phase, based on Sobol sequences; see [5], [7],
- a procedure based on quadratic approximation of one of the equations; see [6].

For each of the tools a proper heuristic has been developed to decide whether using it on a specific box is worthwhile.

It is the opinion of the author that developing such heuristics is crucial for efficiency – and hence applicability – of the interval approach.

Parallelization

Branch-and-prune methods can be parallelized in a relatively simple manner – different boxes can be processed by different threads. The presented solver is implemented using this approach – Intel Threading Building Blocks (TBB) have been used for parallelization.

Usually, operations on a single box are performed by a single thread, but there are exceptions to this rule. In particular, enforcing bound-consistency is an intensive operation and it can be worthwhile to parallelize its execution on a single box.

Box subdivision

Usually, bisection of the longest edge is used in interval algorithms. In [4] and [7] other heuristics have been developed to choose the coordinate for bisection.

During the presentation at SWIM 2015, new results are going to be presented – including heuristics to choose between bi- and multisection, recommended by some authors (e.g., [1], [2]).

References

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