A new class of iterative interval methods for solving linear parametric systems

Lubomir Kolev

Technical University of Sofia
Faculty of Automatica, 1000 Sofia, Bulgaria
lkolev@tu-sofia.bg

Keywords: linear interval parametric systems, parameterized solution, iterative methods

Introduction

The linear interval parametric (LIP) systems considered in the talk include

$$A(p) = A^{(0)} + \sum_{\mu=1}^{m} A^{(\mu)} p_\mu,\ b(p) = b^{(0)} + \sum_{\mu=1}^{m} b^{(\mu)} p_\mu,\ p_\mu = [-1, 1] \quad (1)$$

where $A^{(\mu)}$ $\mu = 0, 1, \ldots, m$ are $n \times n$ real matrices and $b^{(\mu)}$ $\mu = 0, 1, \ldots, m$ are real column vectors. As is well known, the following "interval solutions" to (1) are of interest: outer interval (OI) solution $\mathbf{x}$, interval hull (IH) solution $\mathbf{x}^*$ and inner estimation of the hull (IEH) solution ξ. It should be stressed that all known methods for determining OI or IEH solutions (and, hence, the IH solution) yield the solution sought in the form of an interval vector.

A new type of solution $\mathbf{x}(p)$ to the LIP system (1) (called parameterized or p-solution) has been recently introduced in [1]. It is of the following parametric form

$$\mathbf{x}(p) = c + Lp + \mathbf{s},\ p \in \mathbf{p}$$

where L is a real $n \times n$ matrix while where c and $\mathbf{s}$ are a real and interval symmetric vectors, respectively. The new solution $\mathbf{x}(p)$, $p \in \mathbf{p}$ has a number of useful properties: using it one can determine comparatively narrower $\mathbf{x}$ and ξ as well as small intervals containing the lower and upper ends of each component of $\mathbf{x}^*$. Combined with a constraint satisfaction technique, it permits determination of $\mathbf{x}^*$ as well as the global solution of certain equality-constrained optimization problems [1]. An iterative method for determining $\mathbf{x}(p)$ was suggested in [1] which is obtained by modifying each step of a known iterative method (Reference[11] in [1]) for computing $\mathbf{x}$.

The objective of the present talk is to show that any known iterative method for determining $\mathbf{x}$ can be modified in a unified manner as to produce a corresponding method for determining $\mathbf{x}(p)$. Thus, a whole new class of iterative methods for solving (1) can be constructed.

Iterative scheme

The unified iterative scheme, applicable for any method belonging to the new class, will be illustrated using the fixed-point representation of (1). Hence, the iterative process is

$$x^{(k+1)}(p) = \left(I - A^{(0)} + \sum_{\mu} p_\mu A^{(\mu)}\right) x^{(k)}(p) + \sum_{\mu} b^{(\mu)} p_\mu,\ k \geq 0,\ x^{(0)} = x^0$$

where x^0 is the solution of (3) for $p = 0$. As in [1], it can be shown that each term $x^{(k)}(p)$ in (3) can be enclosed by the linear interval form

$$\mathbf{l}^{(k)}(p) = c^{(k)} + L^{(k)} p + \mathbf{s}^{(k)},\ p \in \mathbf{p}.$$

It can be proved that if the sequence $\mathbf{l}^{(k)}(p)$, $k \geq 1$ is convergent to a limit $\mathbf{l}^{(\infty)}(p)$, then:

(i) the interval vector

$$\mathbf{x} = \mathbf{l}^{(\infty)}(\mathbf{p})$$

($\mathbf{l}^{(\infty)}(\mathbf{p})$ is the range of $\mathbf{l}^{(\infty)}(p)$) is an OI solution to (1);

(ii) the linear interval form

$$\mathbf{x}(p) = \mathbf{l}^{(\infty)}(p) = x^{(0)} + c^{(\infty)} + L^{(\infty)}p + \mathbf{s}^{(\infty)}$$

determines a p-solution to (1);

(iii) the matrix $A(p)$ is non-singular for each $p \in \mathbf{p}$.

The actual iterative method is implemented using Rump's epsilon-inflation technique so the numerical complexity of the method is polynomial.

Detailed analysis shows that each new iterative method based on the use of p-solutions is superior to the corresponding original method as regards conservatism of the results and applicability radius of the methods.

Also, this approach can be applied to implementing new hull consistency algorithms treating several equations simultaneously.

References

[1] L. Kolev, Parameterized solution of linear interval parametric systems, *Applied Mathematics and Computation* 246(11):229-246, 2014.