Yet another method for solving interval linear equations

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Introduction

Solving interval linear equations is one of the basic tasks in interval computations. Let an interval system

$$Ax = b, \quad A \in \mathbf{A},\ b \in \mathbf{b},$$

be given, where $\mathbf{A}$ is an interval matrix and $\mathbf{b}$ an interval vector. The problem is to tightly enclose the solution set defined as

$$\{x \in \mathbb{R}^n \mid \exists A \in \mathbf{A} \exists b \in \mathbf{b} : Ax = b\}.$$

This is a difficult task in general since there is no polynomial-time enclosure method with any fixed accuracy unless $P = NP$. Even to check if the solution set is non-empty is an NP-hard problem.

There are, however, lots of methods known that work well in most of the cases. They differ in computational time and tightness of the resulting enclosures. Many methods use preconditioning, in particular preconditioning by (an approximation of) the midpoint inverse matrix, since a tight enclosure can be computed then. Thus, we will assume that a given interval linear system is preconditioned such that its midpoint matrix is the identity matrix.

Main results

We present a new operator for enclosing the solutions set; see [1]. It generalizes the classical interval Gauss–Seidel operator. Also, based on the new operator and properties of the well-known methods, we propose a new enclosing algorithm, We call it *the magnitude method* since it utilizes the easily computable magnitude (i.e., entrywise the largest absolute value) of the interval hull of the solution set.

The performance of the method depends on how several quantities are computed. If they are computed exactly, then the method yields the interval hull of the preconditioned solution set. On the other hand, even if they are approximated very roughly, the magnitude method performs always as well (w.r.t. tightness) as the Gauss–Seidel iteration method.

We illustrate by numerical examples that our approach overcomes the Gauss–Seidel iteration method with respect to both computational time and sharpness of enclosures. Compared to the INTLAB function `verifylss`, the magnitude method produces always tighter enclosures. Unless the input interval data are very narrow, it also overcomes `verifylss` with respect to computational time.

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References

[1] M. HLADÍK, New operator and method for solving real preconditioned interval linear equations, *SIAM Journal on Numerical Analysis* 52(1):194–206, 2014.