Guaranteed Coverage Assessment of a Robotic Survey with Uncertain Trajectory

Vincent Drevelle

Université de Rennes 1, IRISA, INRIA Rennes-Bretagne Atlantique, Lagadic project
Campus de Beaulieu, 35042, Rennes, France
vincent.drevelle@irisa.fr

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Introduction

Robots are often employed for tasks that consists in covering an given area. Survey missions consists in gathering information (image, relief...) about every point of an area, using embedded sensors like cameras, lidars or sonars. Other tasks such as lawn-mowing or cleaning also involve covering an area with an effector (e.g a blade or a vacuum cleaning system).

In practice, the robot trajectory is known with an uncertainty, which propagates as an uncertainty on the area that has been actually covered during the mission. Assessing the acquired of the survey mission is an important task [1], to ensure there will be no gap when merging the acquired data.

A interval analysis based set-membership approach to computing the explored area with uncertain trajectory will be presented. Single- and multi-robot applications will be demonstrated.

Problem statement

Let us consider a mobile robot equipped with actuators and sensors. The robot is classically represented by the following state equations

$$\left\{ \begin{array}{rcl} \dot{\mathbf{x}}(t) & = & \mathbf{f}\left(\mathbf{x}(t), \mathbf{u}(t)\right) \\ \mathbf{y}(t) & = & \mathbf{g}\left(\mathbf{x}(t)\right), \end{array} \right. \tag{1}$$

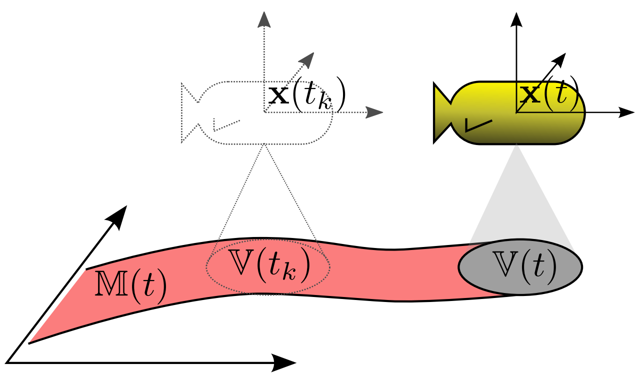
where $\mathbf{x}$ denotes the robot's state vector (e.g. position, velocity...), $\mathbf{u}$ is the input vector and $\mathbf{y}$ is the observation vector. The robot's evolution is modeled by the function $\mathbf{f}$ and $\mathbf{g}$ is the observation function.

The robot uses a sensor for searching or mapping, that can cover a given area. Let $\mathbb{V}$ be the set-valued function that returns the area which is in the robot's field of view at each time, i.e the *visible area* $\mathbb{V}(t)$ at time t. It is defined by the *visibility function* v, such that $v(\mathbf{z}, \mathbf{x}(t))$ is negative iff the point $\mathbf{z}$ is in the range of the sensor for the given the robot state $\mathbf{x}(t)$:

$$\mathbb{V}(t) = \left\{ \mathbf{z} \in \mathbb{R}^2 : v\left(\mathbf{z}, \mathbf{x}(t)\right) \leq 0 \right\}. \tag{2}$$

The *mission's surveyed area* $\mathbb{M}$ is the area that has been covered by the robot's sensor at the end of the exploration mission, i.e from the mission start t_0 to the mission end t_f:

$$\mathbb{M} = \bigcup_{t \in [t_0, t_f]} \mathbb{V}(t). \tag{3}$$



Assuming bounded-error knowledge of $\mathbf{u}$ and $\mathbf{y}$, i.e $\mathbf{u}(t) \in [\mathbf{u}](t)$ and $\mathbf{y}(t) \in [\mathbf{y}](t)$, the problem we want to address consists in computing a bracketing of $\mathbb{M}$ in the form of a set-interval $[\underline{\mathbb{M}}, \overline{\mathbb{M}}]$. The set $\underline{\mathbb{M}}$ is guaranteed to have been covered during the mission, while the complement of $\overline{\mathbb{M}}$ has guaranteedly not been covered.

Let us define the set of admissible trajectories $\mathcal{T} = \{\mathbf{x} : \mathbb{R} \to \mathbb{R}^n \mid \forall t, \, \dot{\mathbf{x}}(t) \in \mathbf{f}(\mathbf{x}(t), [\mathbf{u}](t)), \, \mathbf{g}(\mathbf{x}(t)) \in [\mathbf{y}](t)\}$.

Ideally, $\mathbb{M}$ can be bracketed between the *guaranteed surveyed area* $\mathbb{M}^{\forall} = \left\{\mathbf{z} \in \mathbb{R}^2 \mid \forall \mathbf{x} \in \mathcal{T}, \, \exists t, \, v\left(\mathbf{z}, \mathbf{x}(t)\right) \leq 0\right\}$, and the *possibly surveyed area* $\mathbb{M}^{\exists} = \left\{\mathbf{z} \in \mathbb{R}^2 \mid \exists \mathbf{x} \in \mathcal{T}, \, \exists t, \, v\left(\mathbf{z}, \mathbf{x}(t)\right) \leq 0\right\}$. We thus have $\underline{\mathbb{M}} \subseteq \mathbb{M}^{\forall} \subseteq \mathbb{M} \subseteq \mathbb{M}^{\exists} \subseteq \overline{\mathbb{M}}$.

Approach

Simple approach: union of visible area intervals

A first approach to compute a $[\underline{\mathbb{M}}_{\cup}, \overline{\mathbb{M}}_{\cup}]$ set-interval has been presented in [2]. It consists in first contracting the tube $[\mathbf{x}](t)$ with the constraints of Eq. 1. This is done using the contractor programming approach. Then, the surveyed area interval is obtained as the union of visible area intervals: $[\underline{\mathbb{M}}_{\cup}, \overline{\mathbb{M}}_{\cup}] = \bigcup_{t \in [t_0, t_f]} [\underline{\mathbb{V}}(t), \overline{\mathbb{V}}(t)]$. Symbolic interval arithmetic [3] is used to derive lower and upper bounds of $v(\mathbf{z}, [\mathbf{x}](t))$. A set-inversion method is then employed for surveyed area computation.

Taking robot evolution into account

While being very fast, the previous approach provides very pessimistic bounds for $\mathbb{M}$: Using a tube $[\mathbf{x}](t)$ to represent the set of admissible trajectories $\mathcal{T}$ discards temporal dependancies.

This pessimism is clearly visible when the robot position uncertainty is larger than its sensor field of view. Indeed, in this case, by considering independently the visible areas at each time, it is often not possible to guarantee a non-empty lower-bound for $\mathbb{M}$. However, by considering the admissible trajectories, an non-empty set $\underline{\mathbb{M}}$ can be guaranteed to have been surveyed, thanks to inter-temporal dependency of the robot positions (i.e robot evolution model).

We propose an improved method which yields a tighter set-interval for $\mathbb{M}$. It consists in partitioning and contracting the tube $[\mathbf{x}]$ at given times. This thinner representation of $\mathcal{T}$ yields a smaller set-interval for the mission surveyed area, at the expense of a longer computation time.

Results and comparison of the two methods will be presented on a single robot and a multi-robot test-case.

References

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