Sub-Interval Perturbation Method for Standard Eigenvalue Problem

Nisha Rani Mahato[1] and Snehashish Chakraverty[1,*]

[1] Department of Mathematics,
National Institute of Technology Rourkela, Odisha, India

[*] sne_chak@yahoo.com

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Introduction

Solution of finite element analysis under dynamic condition led to generalized eigenvalue problem. Generally we have crisp values of material properties for structural dynamic problems. As a result of errors in measurements, observations, calculations or due to maintenance induced errors etc. we may have uncertain bounds which may be modeled through interval analysis. So, this paper deals with sub-interval perturbation procedure for computing upper and lower eigenvalue and eigenvector bounds of standard eigenvalue problem with interval parameters.

Few literatures for solving structural dynamics problem based on interval analysis in perturbation approach of structural dynamics are available. In this regard, Alefeld and Herzberger [1] and Moore et al. [2] presented a detailed discussion on interval computations. Qiu et al. [3] proposed an interval perturbation approximating formula for evaluating interval eigenvalues for structures. An inner approximation algorithm has been proposed by Hladik et al. [4] with perturbations belonging to some given interval. For structures with large interval parameters Qiu and Elishakoff [5] proposed a subinterval perturbation for estimating static displacement bound.

As such present section gives the introduction and the next section discusses sub-interval perturbation procedure. In third section, main results based on the developed procedure is considered for a structural standard eigenvalue problem and lastly conclusion is included.

Sub-Interval Perturbation Procedure

Interval matrix of eigenvalue problem has been initially divided into small sub-intervals and then for each interval, unperturbed crisp eigenvalues λ_i^c and eigenvectors x_i^c are obtained from the corresponding center matrices K^c. Generally in structural dynamic problems the stiffness and mass matrices are symmetric in nature. So, by using orthogonality condition of eigenvectors of symmetric matrices and neglecting higher order perturbations, structural system has been perturbed through λ_i^c and x_i^c to obtain eigenvalues $\delta\lambda_i$ and eigenvectors δx_i.

Sub-Interval

Let $A^I = [\underline{a},\ \overline{a}]$ be an interval, then its subintervals may be obtained by dividing the interval into m equal parts with width $(\underline{a}-\overline{a})/m$. In case of an interval matrix K^I of order n, the subinterval matrices are obtained with respect to each element having matrix width $(\underline{K} - \overline{K})/m$. So, the subinterval matrices may be obtained as $K^I = [\underline{K},\ \overline{K}] = \bigcup_{k=1}^{m} K_t^I$ where $K_t^I = [\underline{K}+(t-1)(\underline{K}-\overline{K})/m,\ \underline{K}+t(\underline{K}-\overline{K})/m],\ t = 1,\ 2, ...,\ m$.

Perturbation

Let us consider a standard interval eigenvalue problem

$$K^I x_i^I = \lambda_i^I x_i^I,\ i = 1,\ 2, ...,\ n \tag{1}$$

In term of interval center and radius, equation (1) may be written as

$$(K^c + \delta K)(x_i^c + \delta x_i) = (\lambda_i^c + \delta\lambda_i)(x_i^c + \delta x_i) \tag{2}$$

Then the required first order perturbation of eigenvalues for $\delta\lambda_i = [-\Delta\lambda_i,\ \Delta\lambda_i]$ where $\Delta\lambda_i = (x_i^c)^T \Delta K x_i^c$ may be given by

$$\underline{\lambda}_i = \lambda_i^c - (x_i^c)^T \Delta K x_i^c \tag{3a}$$

$$\overline{\lambda}_i = \lambda_i^c + (x_i^c)^T \Delta K x_i^c \tag{3b}$$

and the first order perturbation of eigenvectors may be written as

$$\underline{x}_i = x_i^c + \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{(x_j^c)^T \Delta K x_i^c}{\lambda_i^c - \lambda_j^c} x_j^c \tag{4a}$$

$$\overline{x}_i = x_i^c - \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{(x_j^c)^T \Delta K x_i^c}{\lambda_i^c - \lambda_j^c} x_j^c \tag{4b}$$

This perturbation procedure is then implemented over each subinterval K_t^I to obtain interval bounds of eigenvalues and eigenvectors.

Main results

We consider here a four degree of freedom spring mass system (Qiu et al. [3]) with mass matrix as crisp identity matrix and interval stiffness matrix $K^I = K^c + \delta K$ such that $\delta K = [-\Delta K, \Delta K]$ with

$$K^c = \begin{bmatrix} 3000 & -2000 & 0 & 0 \\ -2000 & 5000 & -3000 & 0 \\ 0 & -3000 & 7000 & -4000 \\ 0 & 0 & -4000 & 9000 \end{bmatrix} \text{ and } \Delta K = \begin{bmatrix} 25 & 15 & 0 & 0 \\ 15 & 35 & 20 & 0 \\ 0 & 20 & 45 & 25 \\ 0 & 0 & 25 & 55 \end{bmatrix}.$$

Accordingly inner and outer approximations of eigenvalue bounds may be computed for $m = 1$ and sufficiently large m respectively.

- *Inner approximation*: λ_i^I for global (without sub-intervals) stiffness matrix.
- *Outer approximation*: $\lambda_i^I = [min\underline{\lambda}_{it},\ max\overline{\lambda}_{it}]$ where $t = 1,\ 2, ...,\ m$ and m being sufficiently large.

Conclusion

This investigation presents sub-interval perturbation procedure for obtaining inner and outer approximation of eigenvalue bounds for standard interval eigenvalue problems. Accordingly corresponding perturbed eigenvectors are also computed. The perturbation of subintervals may not give exact bounds as higher order perturbations are neglected but provides a tighter first order inner approximation interval bounds with a small deflection from its center. The proposed procedure may also be applied to other practical eigenvalue problems involving interval material properties .

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