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MODELLING AND SOLVING SCHEDULING PROBLEMS USING CONSTRAINT PROGRAMMING

Two worlds

- planning vs. scheduling
  - planning is about finding activities to achieve given goal
  - scheduling is about allocating known activities to limited resources and time
- generic (AI) vs. specific (OR) approaches
  - flexible techniques but bad worst-case runtime (due to search)
  - guaranteed runtime and schedule quality, but inflexible techniques
- theory vs. practice

Talk outline

- Motivation
  - scheduling in practice and in academia
- Constraint programming
  - principles and application in scheduling
- Scheduling model
  - temporal network with alternatives
- System demo
  - FlowOpt project
- Concluding remarks

What you ~~can~~ *will* hear in factory

- "We are different..."
  - means, what you know is useless here
- "Outsiders cannot understand it, it takes a lot of time..."
  - means, you have to listen to us or to spend part of your life here
- "Methods that suite others cannot implemented here..."
  - means, your experience and knowledge are impressive, but you have to start from scratch

Theory vs. practice

- Academy
  - the researcher‘s world consists of resources and their usage
    - “how can I use the resources to get max X and min Y…”
    - “how can I get, using objective metrics, a plan that for the long term, will improve the plant efficiency…”
- Factory planners
  - the planner’s world consists of products and their flow
    - “how can I produce this product now, and this one and that one…”
    - “how can I satisfy Mr. X from sales and Mr. Y from the plant and the customer at the same time, without getting into new troubles…”

Our approach

- Be close to the customer
  - use notions that factory planners are familiar with
- Translate the problem to solving formalism
  - use flexible modelling and solving approach
- Solve the problem to acceptable quality
  - combine heuristics and inference
- Allow customers to modify the solution
  - support interactive changes of solutions

CP

What is CP?

Constraint Programming is a technology for solving combinatorial optimization problems modeled as constraint satisfaction problems:

- a finite set of decision **variables**
- each variable has a finite set of possible values (**domain**)
- combinations of allowed values are restricted by **constraints** (relations between variables)

Solution to a CSP is a complete consistent instantiation of variables.

How does CP work?

How to find a solution to a CSP?

Mainstream **solving approach** combines

inference

- removing values violating constraints
- consistency techniques

with **search**

- trying combinations of values
- depth-first search

Constraint Inference

Example:

- D_a = {1,2}, D_b = {~~1~~,2,3}
- a < b

↳ Value 1 can be safely removed from D_b.

- Constraints are used **actively to remove inconsistencies** from the problem.
  - inconsistency = a value that cannot be in any solution
- The most widely-used technique removes values that violate any constraint until a fixed point is reached (no value violates a single constraints).

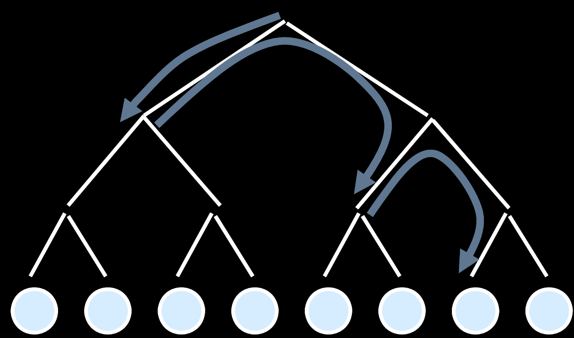
Search / Labeling

Consistency techniques are (usually) incomplete.

↳ **We need a search algorithm to resolve the rest!**

Labeling

- depth-first search
  - assign a value to the variable
  - propagate = make the problem locally consistent
  - backtrack upon failure



- X in 1..5 ≈ $X{=}1 \vee X{=}2 \vee X{=}3 \vee X{=}4 \vee X{=}5$ (enumeration)

In general, search algorithm resolves remaining disjunctions!

- $X{=}1 \vee X{\neq}1$ (step labeling)
- $X{<}3 \vee X{\geq}3$ (domain splitting)
- $X{<}Y \vee X{\geq}Y$ (problem splitting)

How to use CP?

- Using Constraint Programming is less about solving algorithms and more about modeling (similarly to SAT or MIP)
  - constraint modeling = formulation of problem as a CSP
- Moreover, CP directly supports **integration** of **ad-hoc solving techniques** via global constraints and natural expression of **search heuristics** (differently from SAT and MIP).

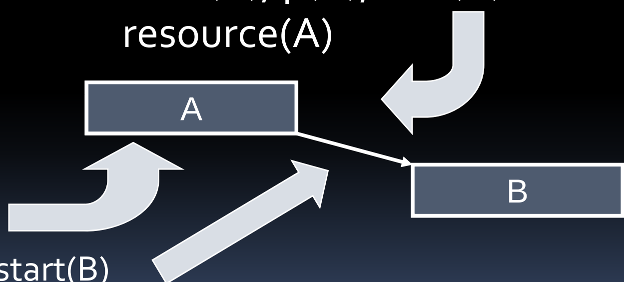
ABC of CBS

Constraint-based scheduling
= Scheduling + Constraint Satisfaction

Variables

a position of activity in time and space

time allocation: start(A), p(A), end(A)
resource allocation: resource(A)



Constraints

Temporal relations:

start(A)+p(A)=end(A)
precedences A«B: end(A) ≤ start(B)

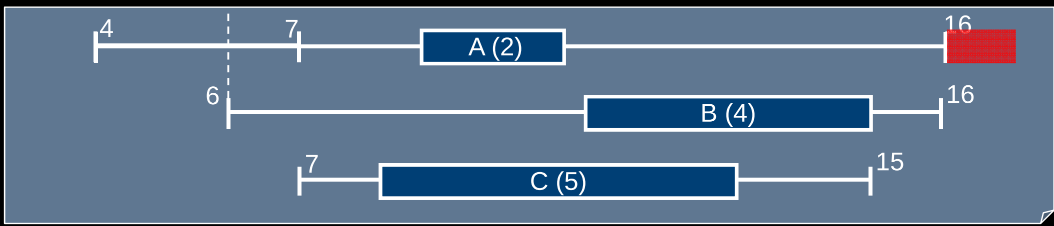
Resource relations:

unary resource A«B ∨ B«A: end(A) ≤ start(B) ∨ end(B) ≤ start(A)

Edge finding

resource inference

- Can we restrict time windows more than using disjunctive constraints?



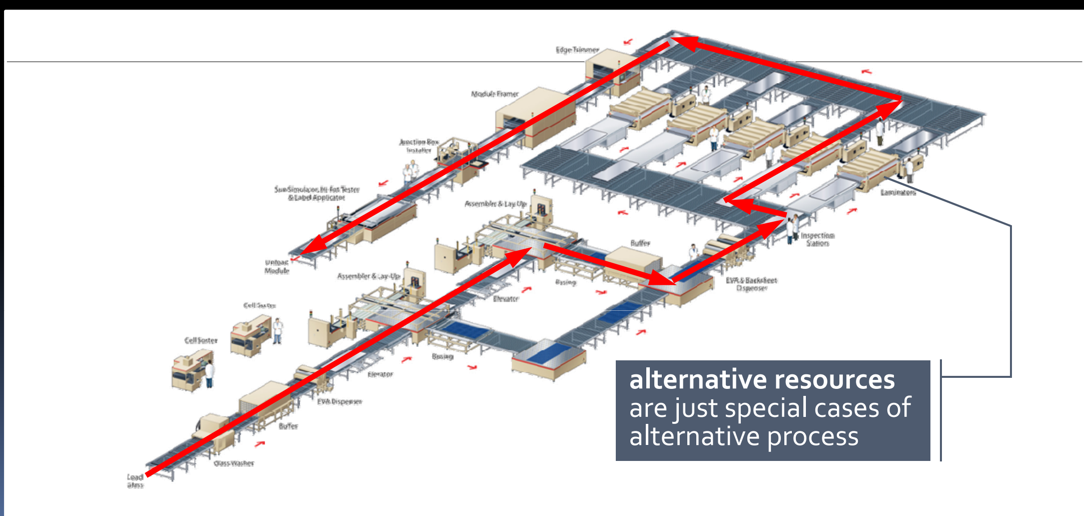
$$p(\Omega \cup \{A\}) > lct(\Omega \cup \{A\}) - est(\Omega) \Rightarrow A \ll \Omega$$
$$A \ll \Omega \Rightarrow end(A) \leq \min\{ lct(\Omega') - p(\Omega') \mid \Omega' \subseteq \Omega \}$$

In practice:

- there are $O(n.2^n)$ pairs (A,Ω) to consider (too many!)
- instead of Ω use so called **task intervals** [X,Y]
  $\{C \mid est(X) \leq est(C) \wedge lct(C) \leq lct(Y)\}$
  ↳ time complexity $O(n^3)$, frequently used incremental algorithm
- there are also $O(n^2)$ and $O(n.\log n)$ algorithms

Our problem

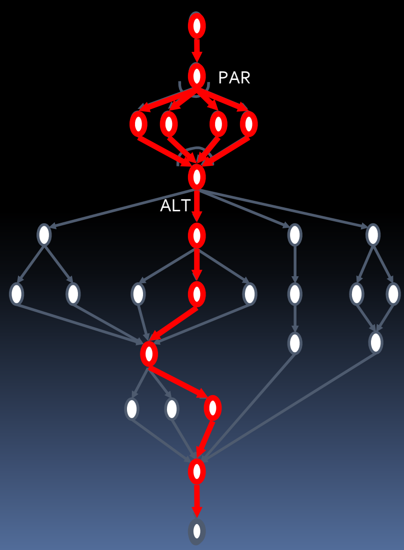
- Real-life production scheduling with alternative process routes and earliness/tardiness cost.
- Involves planning (selection among alternative processes) and scheduling (time and resource allocation).



[FLAIRS 2007]

Conceptual Model

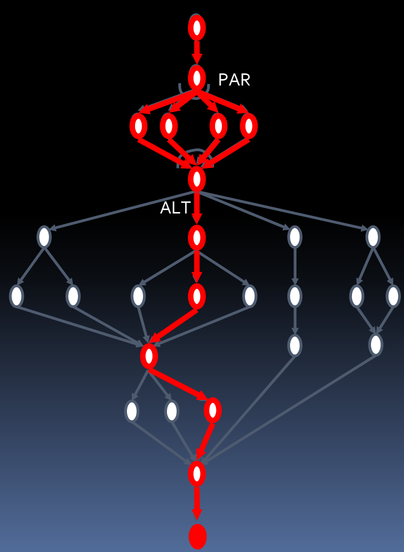
- We model the workflow as a directed acyclic graph called **Temporal network with alternatives** (TNA):
  nodes = operations, arcs = precedence (temporal) relations
  logical dependencies between nodes – **branching relations**.



- The process can split into **parallel branches**, i.e., the nodes on parallel branches are processed in parallel (all must be included).
- The process can select among **alternative branches**, i.e., nodes of exactly one branch are only processed (only one branch is included).
- The **problem** is to select a sub-graph satisfying logical, temporal, and resource constraints.

Problem hardness

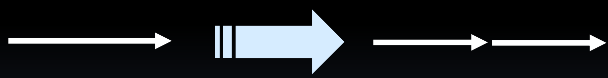
- If all nodes are made invalid (removed from the graph) then we have a **trivial solution** satisfying all the constraints.



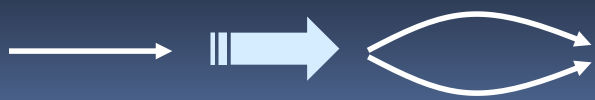
- Assume that **some node must be valid**, i.e., it is specified to be included in TNA.
  - for example, a demand must be fulfilled
- Is it hard to find if it is possible to **select a sub-graph satisfying the branching constraints**?
  - Is it possible to select a process satisfying the demand?
  - The problem is **NP-complete**!!! [FLAIRS 2007].

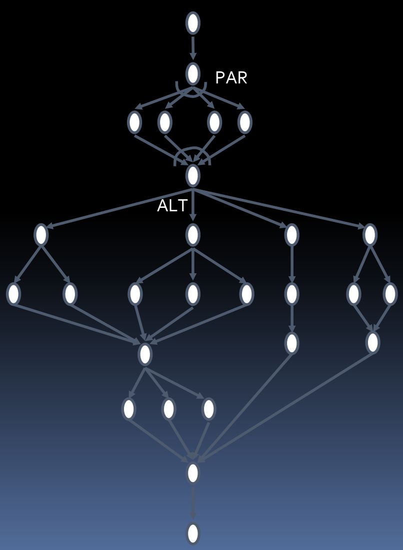
Real processes

- Real manufacturing process networks frequently have a **specific structure**.
  - The process network is built by **decomposing** a „meta-processes“ into more specific processes:
    - **serial decomposition**



    - **parallel/alternative decomposition**

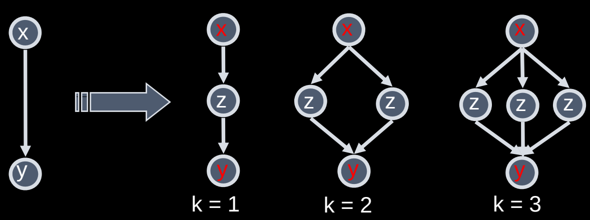




[AIMSA 2008]

Nested graphs

- graphs constructed from a single arc by the following **decomposition operation**:

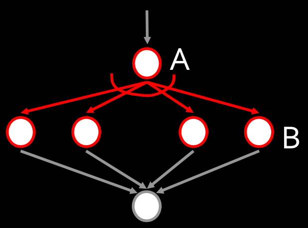


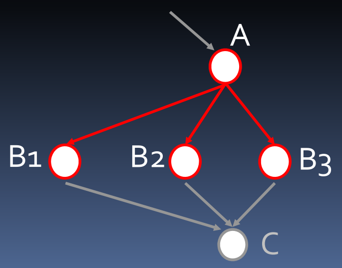
- **Features:**
  - it is a temporal network with alternatives
  - we can algorithmically recognize nested graphs
  - the assignment problem is tractable

[FLAIRS 2007]

Logical constraints

- The path selection problem can be modeled as a **constraint satisfaction problem**.





  - each **node** A is annotated by {0,1} variable V_A
  - each arc (A,B) from a **parallel branching** defines the constraint $V_A = V_B$
  - let arc (A,B1),…, (A,Bk) be all arcs from some **alternative branching**, then $V_A = \Sigma_{i=1,\dots,k} V_{Bi}$

[RAC 2008]

Temporal constraints

- So far we assumed that an arc in the graph describes a **precedence**.
- We can annotate each arc (X,Y) by a **simple temporal constraint** [a,b] with the meaning $\mathbf{a \leq Y\text{-}X \leq b}$.
  - **(Nested) Temporal Network with Alternatives**
- Base constraint model:
  - each **node** A is annotated by a **temporal variable** T_A with a domain $\langle 0,MaxTime \rangle$, where MaxTime is a constant given by the user.
  - Temporal relation [a,b] between nodes X and Y must hold if both nodes are valid!

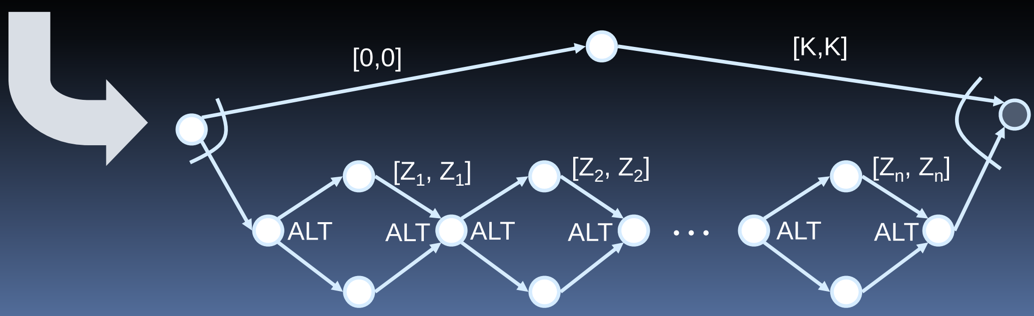
$$\mathbf{V_X * V_Y * (T_X + a) \leq T_Y \wedge V_X * V_Y * (T_Y - b) \leq T_X.}$$

Notes:

- $V_X = 0 \vee V_Y = 0 \rightarrow 0 \leq T_Y \wedge 0 \leq T_X$
- $V_X = V_Y = 1 \rightarrow (T_X + a) \leq T_Y \wedge (T_Y - b) \leq T_X.$
- The above temporal constraint does not assume the type of branching!

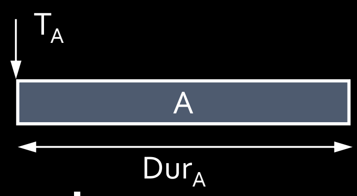
Temporal hardness

- **Is it possible to achieve global consistency of temporal relations in nested graphs?**
- Unfortunately, the problem is **NP-complete** ☹
  - **Subset sum problem** can be converted to temporal feasibility of nested graphs.
  - Let Z_i, i = 1,…,n be integers, is there a subset S of {1,…,n} such that $\Sigma_{i \in S} Z_i = K$?



Resource constraints

- **standard scheduling model**
  - start time variable: $\mathbf{T_A}$
  - duration variable: $\mathbf{Dur_A}$



- unary (disjunctive) **resource constraints**
  - two operations allocated to the same resource do not overlap in time

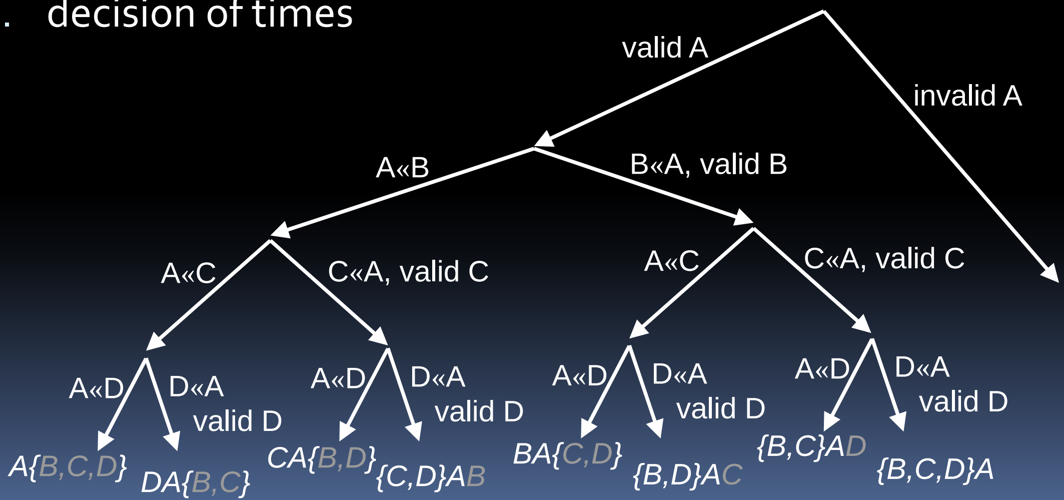
$$V_X * V_Y * (T_X + Dur_X) \leq T_Y \vee V_X * V_Y * (T_Y + Dur_Y) \leq T_X$$

  - or, we can use **existing global constraints** modeling unary resource (edge-finding, not-first/not-last, etc. inference techniques) extended to optional operations
    - (in)valid operations: $\mathbf{Val_A = 1 \Leftrightarrow Dur_A > 0}$

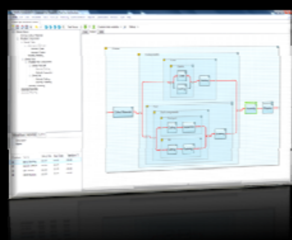
[CSCLP 2008]

Branching Strategy

1. ordering of activities in resources (with activity selection)
   - select some activity (earliest start combined with other criteria)
   - make the activity valid
   - decide its position in the resource (from start)
2. decision of times



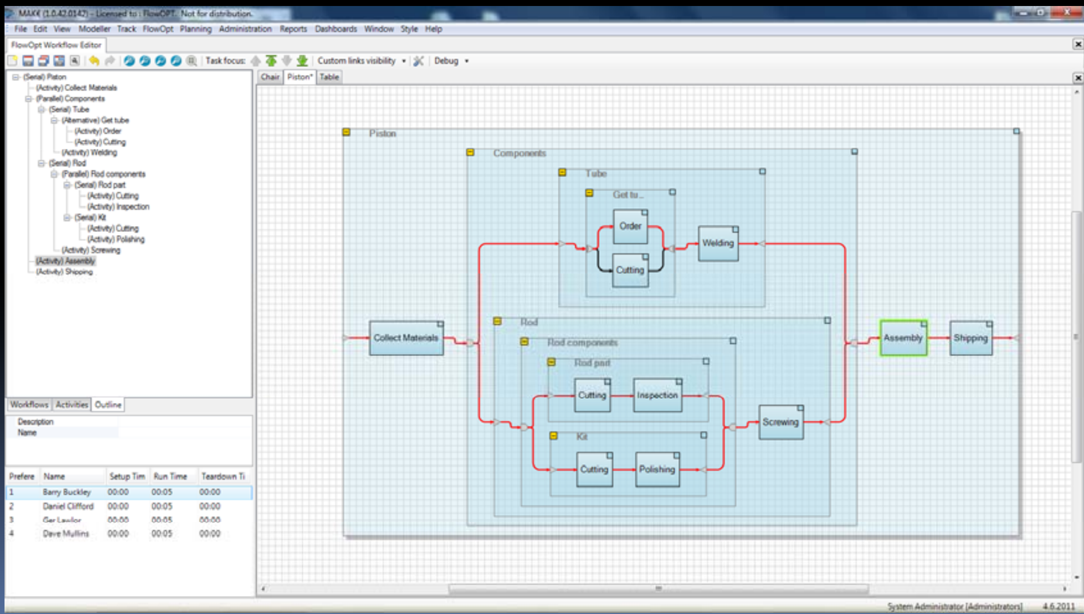
Demo



- **FlowOpt** tools build on top of enterprise optimisation system MAK€ for SMEs
  - build-to-order (engineer-to-order) production
  - on-time-in-full objective (earliness/tardiness)
- What will you see?
  - interactive graphical design of workflows
  - creating and scheduling custom orders
  - visualisation and modification of schedules
  - schedule analysis

Workflow editor

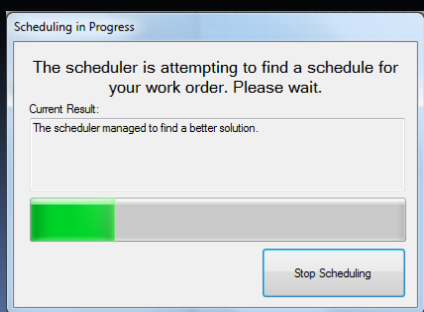
- top-down and bottom up approach to design **nested workflows**
- supports **extra logical** (mutual exclusion,…) and **temporal** (synchronization,…) **constraints**



Optimiser

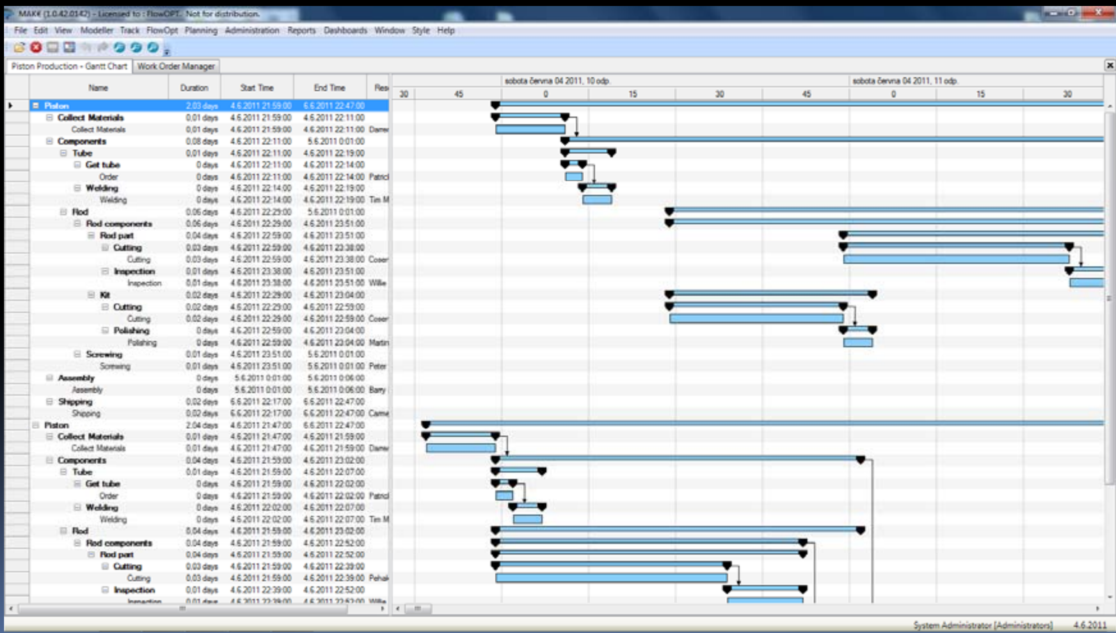
a fully **automated scheduler** that takes description of workflows for ordered products and generates a schedule

- implemented in ILOG CP Optimiser (OPL Studio)
- branch-and-bound optimisation (earliness and lateness costs and cost for alternatives are assumed)



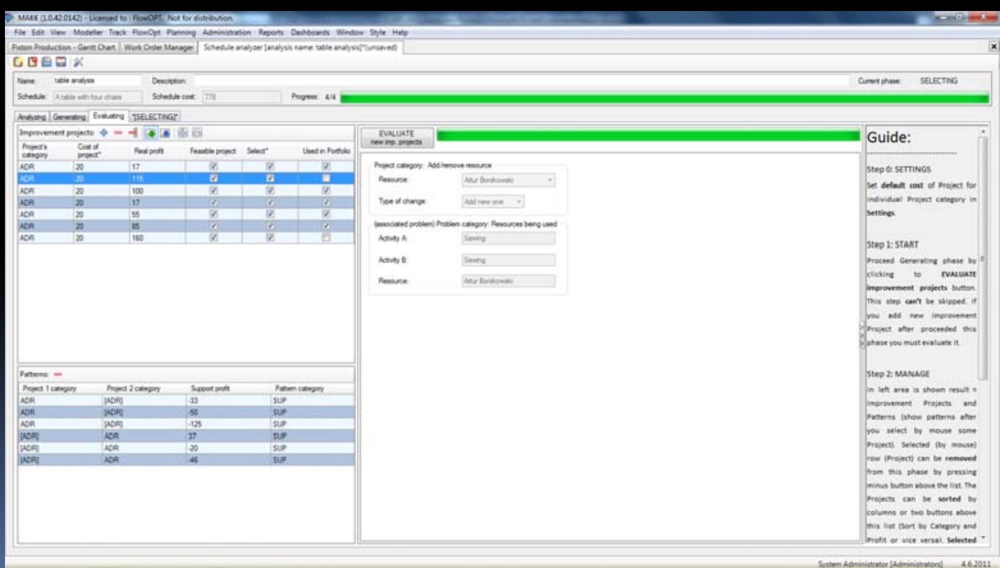
Gantt Viever

- visualization and modification of schedules

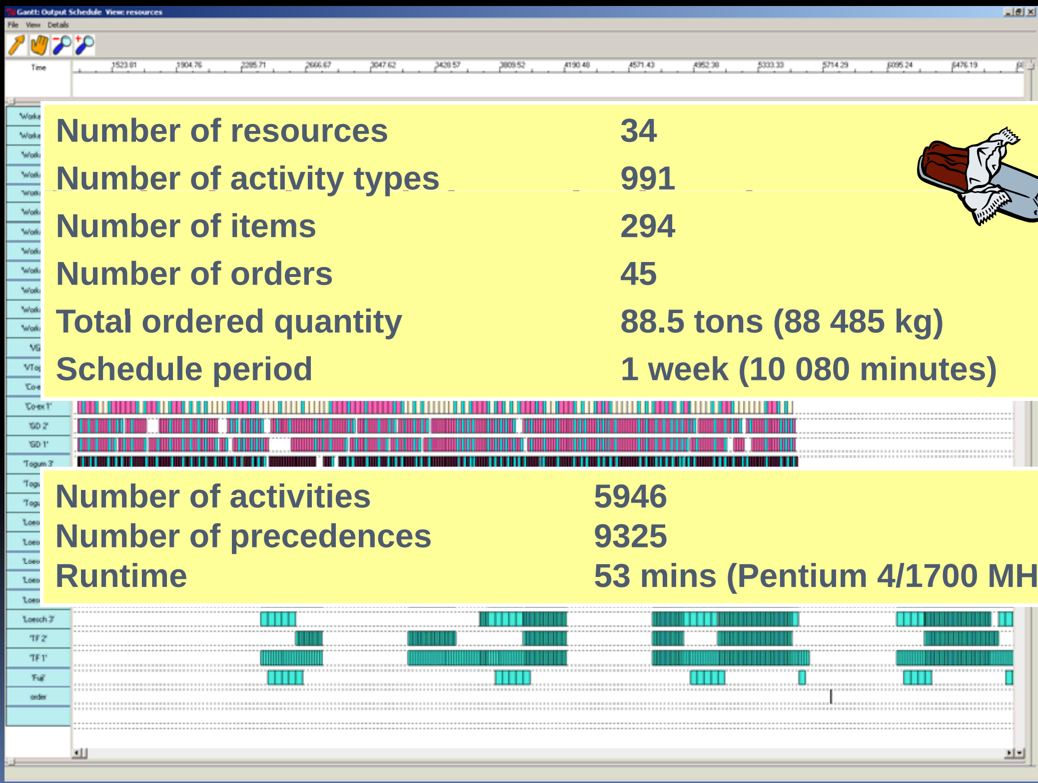


Analyser

- analysis of problems in schedules (late deliveries) and suggestions for enterprise improvements (buying a new resource)



Some results



Number of resources	34
Number of activity types	991
Number of items	294
Number of orders	45
Total ordered quantity	88.5 tons (88 485 kg)
Schedule period	1 week (10 080 minutes)

Number of activities	5946
Number of precedences	9325
Runtime	53 mins (Pentium 4/1700 MHz)

Summary

- Scheduling is not only mathematics but first of all a knowledge handling process.
  - how to capture real knowledge?
  - how to represent it formally so the user can verify it and update it?
  - how to exploit mathematical methods when real-life constraints are present?
- **The art of real-life scheduling is to deliver a plan which is good enough and fast enough.**
  - good enough – the user cannot improve it in reasonable time
  - fast enough – depends on the plant dynamics. One hour can be too late for one plant and very fast to another.