

NMAG403 - Combinatorics

October 06, 2023 – Basics

In class problems

1. Prove a slightly stronger version of Menger theorem: For every positive integer k , a graph is vertex- k -connected if and only if every two distinct **non-adjacent** vertices are connected by at least k internally vertex-disjoint paths.
2. Prove that for every positive integer k , a graph with at least $k + 1$ vertices is vertex- k -connected if and only if every two k -element sets X, Y of vertices are connected by k completely disjoint paths.
3. Prove that in a vertex-2-connected graph, for any two vertices, there is a cycle that passes through both of them.
4. Prove that for every positive integer k , every k vertices in a vertex- k -connected graph lie on a cycle in the graph.
5. Prove that any connected graph G with at least 2 vertices contains two different vertices u, v such that both $G - u$ and $G - v$ are connected.
6. Does there exist a graph with at least 2 vertices such that every two distinct vertices have different degrees?
7. Prove that every planar vertex-2-connected graph has an $s - t$ -numbering (an orientation with a single source and a single sink which allows an upward plane drawing).
8. Prove Brooks theorem which says that for every graph G , $\chi(G) \leq \Delta(G)$ holds true, unless G is a complete graph or a cycle of odd length.