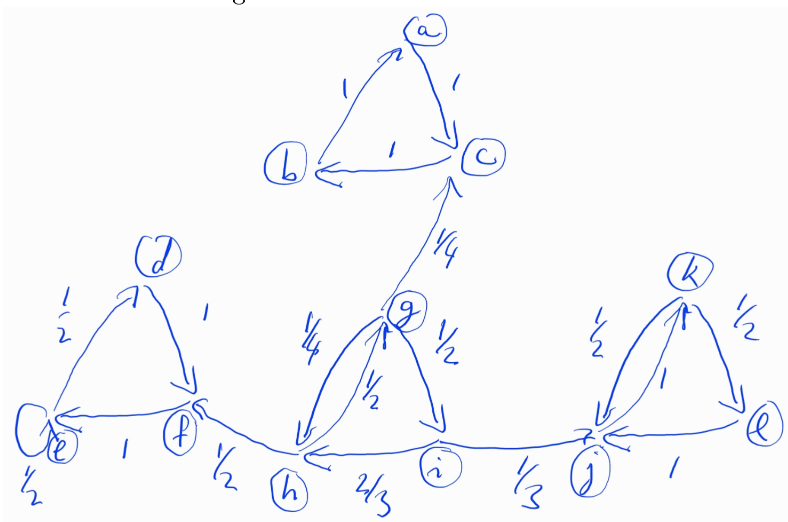
# Credit test – B – Prob. & Stat. 2 — Dec 8, 2022

**1.** (20 points)
Consider the Markov chain on the figure.




(a) Identify the transient and recurrent states. Determine the equivalence classes of $\leftrightarrow$. Explain which of the recurrent classes are periodic.

(b) Does the distribution converge to a stationary distribution, if we start in state $a$? If so, what is the stationary distribution?

(c) Does the distribution converge to a stationary distribution, if we start in state $j$? If so, what is the stationary distribution?

(d) If the process starts in state $h$, what is the probability it ever reaches state $f$?

(e) If the process starts in state $h$, what is the expected time to reach a recurrent state?

(f) Assume the process starts in state $j$, but we observe it after it (approximately) reaches stationary distribution. Find the probability that the state does not change in the next two steps. In other words, approximate $P(X_{1002} = X_{1000} \mid X_0 = j)$.

**2.** (15 points)
The number of mosquito bites per hour follows the Poisson distribution with parameter $\vartheta$. We assume that $\vartheta$ is a value of random variable $\Theta$ that follows $Exp(1/8)$ (the exponential distribution with expectation 8).

(a) In one hour we got bitten 9 times. What is our posterior distribution for $\Theta$? What is MAP estimate and what an LMS estimate?

(b) We did another two measurements (two hours of counting mosquitos): we got 8 and 9 bites per hour. After the three measurements, repeat the previous part.

In your answers you may use the primitive function for any function of type $x^a e^{-bx}$, thas is $F_{a,b}(x) := \int x^a e^{-bx} dx$. You don't need to find a formula for $F_{a,b}$ (it is straightforward to get the formula by repeated per-partes integration, but it is not the point of this problem).

**3.** (15 points)
We are observing arrivals of trams before the school building (direction to Narodni metro station). We assume that tram #22 arrivals are described by a Poisson process with average waiting time 10 minutes and tram #23 arrivals by an independent Poisson process with average waiting time 30 minutes.

(a) What is the probability we have to wait for more than 10 minutes?

(b) We decide to go by the third tram. What is the expected waiting time?

(c) What is the probability the first tram is #23?

(d) What is the probability of five 22s and one 23 during a particular half-hour wait?

(e) Given there were three trams in ten minutes, what is the probability that one of them was 23?