Combinatorics and Graph Theory III - 2020/21

Series 8

1. Determine the choosability of the following graphs: complete graph without one edge, $K_{2,3}$, $K_{2,22}$, $K_{3,3}$, $K_{3,33}$, C_{2n}.

2. * Determine the choosability of the graph $\Theta_{2,2,2m}$ - two vertices connected by three paths of lengths 2, 2, and $2m$ (for integer m).

3. Find the graph G tž. $\chi_l(G) > \chi(G)$ and $|V(G)| + |E(G)|$ is the smallest possible/as small as you can.

4. Show that $\chi_l(K_{n,n^n}) = n + 1$.

5. Show that each planar triangle-free graph has a choosability of at most 4.

6. * Find a planar triange-free graph whose choosability is greater than 3.

7. Let G be a connected graph of maximum degree Δ. If G is not a clique or an odd cycle, then $\chi_l(G) \leq \Delta$.

8. Show that $\chi_l(G) + \chi_l(\bar{G}) \leq n + 1$ holds for any graph G with n vertices and its complement $\bar{G}$.

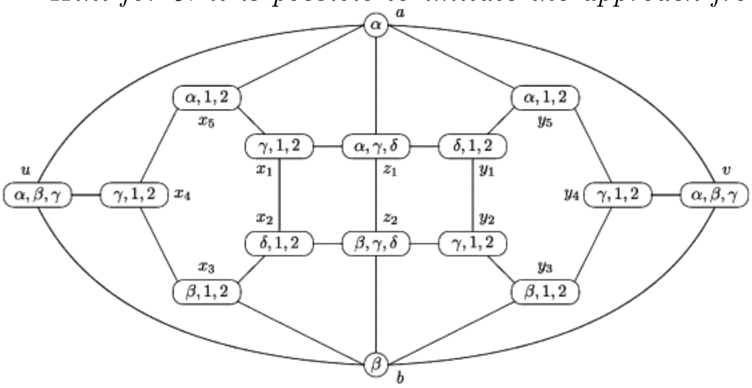
9. * Let G be a connected graph of the minimum degree at least 2. Show that G is 2-choosable if and only if G is either an even cycle or a union of three paths of even length with common ends such that at least two of these paths are of length 2.

10. Let G be a planar graph with the outer face bounded by the induced cycle K. If each vertex G has at most two neighbors in K, then each 5-coloring of K can be extended to a 5-coloring of G.

Hint for 4: one inequality was done in class.

Hint for 5: it suffices to use degeneratedness.

Hint for 6: it is possible to imitate the approach from class,



Hint for 7: follow the proof of Brooks' theorem: find a vertex v and its two nonadjacent neighbors u, w. Then find a spanning tree of the graph with root v, where u, w are leafs.

Hint for 8: it suffices to use degeneratedness.

Hint for 10: use Thomassen's theorem in the stronger version.