Grupová souvislost

Γ — abel. grupa, G graf.

Df G je $\boxed{\Gamma\text{-souvislý}}$ $\iff$ $\forall b: V(G) \to \Gamma$ (t.ž. $\sum b = 0$) $\exists \varphi \dots$ NZ Γ-tok & $\delta\varphi = b$

$u \to v$ $\boxed{\text{okraj}}$

$$\boxed{\delta\varphi(v) = \sum \varphi(vx) - \sum \varphi(xv)}$$

$0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad -1$

($\Downarrow$ G je souvislý) $\qquad \Downarrow$ $\boxed{\exists \text{ NZ } \Gamma\text{-tok}}$

Věta

1) G je Γ-souvislý

2) $\forall f: E(G) \to \Gamma \ \exists \varphi: E(G) \to \Gamma: \varphi$ je Γ-tok & $\forall e \ \varphi(e) \neq f(e)$

3) $\forall b: V(G) \to \Gamma \ \forall f: E(G) \to \Gamma \ \exists \varphi \ \dots\dots$

$\mathbb{Z}_k$-orientace = "± 1 flow in $\mathbb{Z}_k$" … orientace hran t.ž. $\forall v$ $\deg^+(v) \equiv \deg^-(v) \pmod k$

Hypotéza (Tutte) $\forall G$: G je hranově 4-souvislý $\Rightarrow$ G má nt 3-tok $\mathbb{Z}_3$-tok

Hypo (Jaeger) $\exists k$ — " — k — " —

Věta (Thomassen) $\forall G$ hr. 8-souvislý $\Rightarrow$ G je $\mathbb{Z}_3$-souvislý

(LTWZ) — " — 6-souvislý $\Rightarrow$ — " —

Df Dán G, $\beta: V(G) \to \mathbb{Z}_k \setminus \{0, \dots, k-1\}$ obvykl. Def. $\tau: V(G) \to \{0, \pm 1, \dots, \pm k\}$:
$\forall x \in V$ $\tau(x) \equiv \beta(x) \mod k$
$\equiv \deg(x) \mod 2$

$\boxed{k=3}$

deg \ β	0	1	2
0	0	-2	2
1	± 3	1	-1

Theorem 3.1. *Let G be a graph with Z_k-boundary β. Let z_0 be a vertex of $V(G)$, and let D_{z_0} be a pre-orientation of $E(z_0)$. Let $V_0 = \{v \in V(G) - z_0 \mid \tau(v) = 0\}$. If $V_0 \neq \emptyset$, we let v_0 be a vertex of V_0 with smallest degree. Assume that*

$= 4 + |\tau(z_0)|$ $\boxed{k=3}$

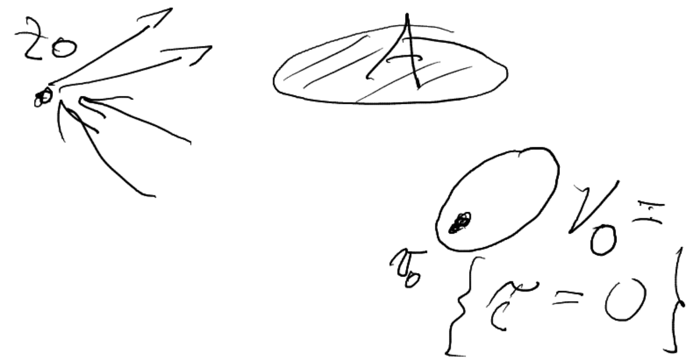
(i) $|V(G)| \geqslant 3$.

(ii) $d(z_0) \leqslant (2k-2) + |\tau(z_0)|$, *and the edges incident with z_0 are pre-directed such that* $d^+(z_0) - d^-(z_0) \equiv \beta(z_0) \pmod{k}$.

(iii) $d(A) \geqslant (2k-2) + |\tau(A)|$ *for each nonempty vertex subset A which does not contain z_0 and which satisfies the conditions that $A \neq \{v_0\}$ and $|V(G) \backslash A| > 1$.*

Then the pre-orientation D_{z_0} of $E(z_0)$ can be extended to a β-orientation D of the entire graph G, that is, for each vertex x of $V(G)$,

$$\boxed{d^+(x) - d^-(x) \equiv \beta(x) \pmod{k}.}$$



Dk – sporem

$M = \{ (G, \beta, z_0)$: věta nepl. & $|V(G)| + |E(G) \setminus z_0|$ min. $\}$

Claim 1. *If $A \subseteq V(G) - z_0$ is a vertex subset such that $1 < |A| < |V(G) - z_0|$, then*

$$d(A) \geqslant 2k + \left|\tau(A)\right|.$$

Claim 2. $V_0 = \emptyset$.

Claim 3. *$G - z_0$ is connected, and $d(z_0) \geqslant k$.*

Claim 4. *For any two distinct vertices $x, y \in V(G) - z_0$, we have $\tau(x)\tau(y) > 0$.*

Claim 5. $V(G) - z_0 = V^+$ *or* $V(G) - z_0 = V^-$.