(1)

$$\int \frac{2\cos x \sin x - \cos^3 x + \cos x}{(\sin x - 1)^2 (\sin^2 x + 5)}\, dx$$

existuje pro $\sin x < 1$ ... $x \in \left(\frac{\pi}{2}, \frac{5}{2}\pi\right) + 2k\pi$

$s = \sin x$

$ds = \cos x \, dx$

$$= \int \frac{2u + u^2}{(u-1)^2(u^2+5)}\, du$$

tento integrál ale pro $u \in (-\infty, 1)$ a $u \in (1, \infty)$

$$\underbrace{}_{\parallel}$$

$$\frac{Au + B}{u^2+5} + \frac{C}{(u-1)^2} + \frac{D}{u-1} \quad (*) \rightarrow$$

$$= \int -\frac{1}{4} \cdot \frac{2u}{u^2+5} + \frac{1}{2} \frac{1}{(u-1)^2} + \frac{1}{2} \frac{1}{u-1}\, du$$

$$= -\frac{1}{4} \ln(u^2+5) - \frac{1}{2} \frac{1}{u-1} + \frac{1}{2} \ln(u-1) + C$$

$$= \boxed{-\frac{1}{4} \ln(\sin^2 + 5) - \frac{1}{2} \frac{1}{\sin x - 1} + \frac{1}{2} \ln(1 - \sin x) + C}$$

$$x^2+2x = (Ax+B)(x-1)^2 + C(x^2+5) + D(x^2+5)(x-1)$$

$x=1$: $3 = C\cdot 6 \Rightarrow C = \frac{1}{2}$

$x=0$ $0 = B + 5C - 5D$

$x=\sqrt{-5}$ $-5+2\sqrt{5}\,i = (A\sqrt{5}\,i + B)\underbrace{(\sqrt{5}\,i-1)^2}_{= (-2\sqrt{5}\,i - 4)}$

$$= A\cdot(+10) + B\cdot(-4) + i\left(A\cdot(-4\sqrt{5}) + B(-2\sqrt{5})\right)$$

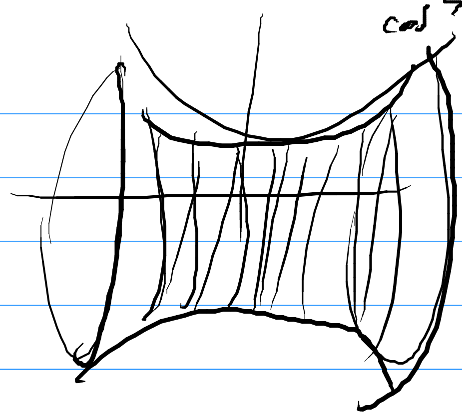
$$\Rightarrow \quad 10A - 4B = -5$$
$$-4A - 2B = 2 \quad |\cdot 2 \,/\, - \quad \Rightarrow 18A = -9$$

$$A = -\frac{1}{2}$$

$$B = -2A - 1 = 0$$

$$D = C + \frac{B}{5} = \frac{1}{2}$$

2



$$V = \int_{-a}^{a} \pi f^2$$

$$= \int_{-a}^{a} \frac{\pi}{4}\left(e^{2x} + 2 + e^{-2x}\right)$$

$$= \frac{\pi}{4}\left[\frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2}\right]_{-a}^{a}$$

$$= \underline{\underline{\frac{\pi}{4}\left(e^{2a} - e^{-2a} + 4a\right)}}$$

$$S = \int_{-a}^{a} 2\pi f \sqrt{1 + f'^2}$$

$$f' = \frac{e^{x} - e^{-x}}{2}$$

$$1 + f'^2 = \frac{e^{2x} - 2 + e^{-2x}}{4} + \frac{4}{4}$$

$$= \left(\frac{e^{x} + e^{-x}}{2}\right)^2$$

$$= \int_{-a}^{a} 2\pi \left(\frac{e^{x} + e^{-x}}{2}\right)^2$$

$$= \ldots = \underline{\underline{2\pi\left(e^{2a} - e^{-2a} + 4a\right)}}$$

(3) $f(x,y,z) = x+y+z$

$M = \{(x,y,z) : x^2+y^2+z^2 = 1 \quad x+2y+3z = 0\}$ $\to g_1 = x^2+y^2+z^2-1$

$\to g_2 = x+2y+3z$

$\nabla g_1 = (2x, 2y, 2z)$

$\nabla g_2 = (1,2,3) \neq 0$

LZ $\Leftrightarrow$ $x = t,\ y = 2t,\ z = 3t$

$1 \cdot t + 2 \cdot 2t + 3 \cdot 3t = 0$

$\Rightarrow t = 0$

$\Rightarrow x = y = z = 0$

nemožné

$\nabla g_1, \nabla g_2$ jsou LN

$\Downarrow$

v bodě extrému je $\nabla f = \lambda \nabla g_1 + \mu \nabla g_2$

$$\left.\begin{aligned} 1 &= 2\lambda x + \mu \\ 1 &= 2\lambda y + 2\mu \\ 1 &= 2\lambda z + 3\mu \end{aligned}\right\} \Rightarrow \text{(1)} \quad x = \frac{1-\mu}{2\lambda} \quad y = \frac{1-2\mu}{2\lambda} \quad z = \frac{1-3\mu}{2\lambda}$$

(2) $\lambda = 0$
$\mu = 1$
$\mu = \frac{1}{2}$ $\}$

$$0 = x+2y+3z = \frac{(1-\mu)+2(1-2\mu)+3(1-3\mu)}{2\lambda}$$

$$0 = \frac{6 - 14\mu}{2\lambda} \Rightarrow \mu = \frac{6}{14} = \frac{3}{7}$$

$$x = \frac{2}{7}\cdot\frac{1}{\lambda}, \quad y = \frac{1}{14}\cdot\frac{1}{\lambda}, \quad z = -\frac{1}{7}\cdot\frac{1}{\lambda}$$

$$1 = x^2 + y^2 + z^2 = \frac{1}{\lambda^2}\cdot\frac{4 + \frac{1}{4} + 1}{49}$$

$$= \frac{1}{\lambda^2}\cdot\frac{21}{4\cdot 49}$$

$$\lambda = \pm\frac{\sqrt{21}}{14}$$

__min:__ $x = -\frac{4}{\sqrt{21}}, \quad y = -\frac{1}{\sqrt{21}}, \quad z = \frac{2}{\sqrt{21}}$

$f = -\frac{3}{\sqrt{21}}$

__max__ $x = \frac{4}{\sqrt{21}}, \quad y = \frac{1}{\sqrt{21}}, \quad z = -\frac{2}{\sqrt{21}}$

$f = \frac{3}{\sqrt{21}}$

__Proč to funguje?__ 1) Dle Lagr. věty jsou [illegible] všechny lok. extrémy f na M.

2) f je spoj. fce na kompaktu $\Rightarrow$ nab. glob. min a max

$\Rightarrow$ jsou to ty

M kompaktní:

- M je omez. $\left(x^2+y^2+z^2=1 \Rightarrow x,y,z \in [-1,1]\right)$
- M je uz.: $M = \underbrace{\underbrace{g_1^{-1}(\{0\})}_{\text{uz.}} \cap \underbrace{g_2^{-1}(\{0\})}_{\text{uz.}}}_{\text{uz.}}$

uz. ⇓