Name: .... **April 6**

Exercise 1 (5 points) Write down the Gramm–Schmidt orthonormalization algorithm.

Exercise 2 (10 points) Use Gramm–Schmidt algorithm on vectors $u_1 = (0,3,0,4,0)^T, u_2 = (0,0,0,5,0)^T, u_3 = (2,1,2,0,0,)^T, u_4 = (4,5,4,9,7)^T$. Then extend the resulting basis by a vector v to a basis of the space $\mathbb{R}^5$.

Exercise 3 (5 points) Find the distance between the point $[0,0,2,0,0]$ and the hyperplane spanned by vectors $u_1, \ldots, u_4$ from Exercise 2.

Exercise 4 (5 points) Find an inverse matrix A^{-1} to matrix

$$A = \begin{pmatrix} 0 & 3 & 4 & 0 \\ 0 & -4 & 3 & 0 \\ 6 & 0 & 0 & -2 \\ 2 & 0 & 0 & 6 \end{pmatrix}.$$

Exercise 5 (5 points) Find a projection of vector $(2,2,1,5)^T$ onto a row space of matrix

$$A = \begin{pmatrix} 0 & \frac{3}{5} & \frac{4}{5} & 0 \\ 0 & \frac{-4}{5} & \frac{3}{5} & 0 \\ \frac{\sqrt{2}}{2} & 0 & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}.$$

Exercise 6 (5 points) Give the definition of norm.

Exercise 7 (5 points) Prove that ℓ_1-norm, given by $\|(x)\|_1 = \sum_{i=1}^{d} |x_i|$ (in a d-dimensional real vector space) is a norm.